Carlo Emilio BONFERRONI
b. 28 January 1892 - d. 18 August 1960

Summary. His name is attached to the Bonferroni Inequalities which facilitate the treatment of statistical dependence. Improvements to the inequalities have generated a large literature.

Carlo Emilio Bonferroni was born in 1892 at Bergamo, a university town in northern Italy. He studied conducting and piano at the conservatory in Torino (Turin), and then studied for the degree of *laurea* in mathematics in Torino under Peano and Segre. He spent a year broadening his education in Wien (Vienna) at the University, and in Zürich at the Eidgenössicher Technischen Hochschule. During the 1914-1918 war he served as an officer in the engineers. He became *incaricato* (assistant professor) at the Turin Polytechnic, and then in 1923 took up the chair of financial mathematics at the Economics Institute in Bari where he was also Rector for 7 years. In 1933 he transferred to Firenze (Florence) where he held his chair until his death in 1960. He was Dean of his Faculty for five years. He received honours from within his own country, but the only one from outside was from the Hungarian Statistical Society.

The obituary of him by Pagni lists his works under three main headings: actuarial mathematics (16 articles, 1 book); probability and statistical mathematics (30, 1); analysis, geometry and rational mechanics (13, 0). His name is familiar in the statistical world through his 1936 paper in which the Bonferroni Inequalities first appear. If p_i is the probability of having characteristic i, p_{ij} the probability of having i and j and so on then he introduces the notation $S_0 = 1$, $S_1 = \sum p_i$, $S_2 = \sum p_{ij}$, $S_3 = \sum p_{ijh}$, ... Then writing P_r for the probability of exactly r events

$$P_0 \leq 1, P_0 \geq 1 - S_1, P_0 \leq 1 - S_1 + S_2, P_0 \geq 1 - S_1 + S_2 - S_3$$

and so on. The first of these inequalities is due to Boole (q.v.) in 1854. It is the highlighting of Boole's Inequality by Francesco Paolo Cantelli (1875-1966) as a tool for treating statistical dependence at the International Congress of Mathematicians held at Bologna in 1928, which Bonferroni attended, that may have led Bonferroni to produce the elegant pattern of the other members of the sequence. Attribution to Boole is in fact made on pp. 4 and 25 of Bonferroni's paper , in which the inequalities are justified in "symbolic" fashion". Similar ideas using S_k's were pursued by Károly Jordan (q.v.) and Henri Poincaré.

The inequalities achieved their popularity through a book of Maurice Fréchet (q.v), a frequent correspondent of Cantelli's, of 1940; and William Feller's celebrated *An Introduction to Probability Theory and its Applications,*, Vol.1, first published in 1950, which was probably the source for most English speaking readers, It is interesting to note that he cites only the monograph by Fréchet.

The inequalities have given rise to a large literature. The first two have been used in particular in simultaneous statistical inference. The method known as Bonferroni adjustment usually relies only on Boole's Inequality.

Bonferroni's 1936 article uses the classical definition of probability, in terms of a finite sample space of equally likely events, usually attributed to Laplace (q.v.). His notion of probability was not, however, confined to this. In his inaugural address for the academic year (1924-25) published in 1927 he clearly states:

> A weight is determined directly by a balance. And a probability, how is that determined? What is, so to say, the probability balance? It is the study of frequencies which gives rise to a specific probability (p. 32)

Then he moves on to consider long run frequency in more detail. On p. 35 he specifically denies that subjective probability is amenable to mathematical analysis.

After these papers he moved away from writing on the foundations of probability. A reason for this change of direction could have been the appearance of the work of von Mises (q.v.) now often regarded as a landmark in the development of the frequentist view.

Bonferroni also worked in a number of other statistical areas. A competitor to the well-known Gini (q.v.) index of concentration, Bonferroni's concentration index is designed to measure income inequality. Let $x_{(i)}$ be the observed ith order statistic in a sample of size n, so that $x_{(i-1)} \leq x_{(i)} \quad (i = 2, \ldots, n)$. Define $m_i, i = 1, 2, ..., n$, as the sample partial means, so that $m = m_n$ is the ordinary sample mean, by:

$$m_i = \frac{1}{i} \sum_{j=1}^{i} x_{(j)}, \; i = 1, 2, ...n.$$

Then Bonferroni's index B_n is

$$B_n = 1 - \frac{1}{n-1} \sum_{i=1}^{n-1} \frac{m_i}{m}$$

He was also interested in "algebraic" means which he treats extensively in his textbook, *Elementi di Statistica Generale.* The algebraic mean M_p of order p is $\sqrt[p]{\frac{x_1^p + \ldots + x_2^p}{n}}$. He published on the properties of a generalisation of this M_{p+q} defined as $\sqrt[p+q]{\frac{x_1^p x_2^q + x_1^q x_2^p + \ldots}{n(n-1)}}$ and similarly for higher orders.

One reason for his current lack of recognition may be the fact that his books were never properly disseminated. Apart from *Elementi di analisi matematica*, and that only in its last edition of 1957, and a smaller research monograph: *Sulla correlazione e sulla connessione* (1942), they probably do not exist in typeset versions. One of the volumes, *Elementi di Statistica Generale* was reprinted in facsimile after his death at the instigation of the Faculty of Economics of University of Firenze; bound with it is a memoir by de Finetti. The reason his books were never properly typeset is that he believed that books were too expensive for students to buy, and so to hold costs down he handwrote his teaching material, and had the books printed from that version. (This pattern is reasserting itself for the same reasons with the aid of electronic publishing.) They run to hundreds of pages, neat and almost correction free. His articles have a clear explanatory nature. He was someone with a genuine interest in communicating his ideas to his audience.

References

Benedetti, C. (1982). Carlo Emilio Bonferroni (1892-1960). *Metron*, **40**, N.3-4, 1-36.

Bonferroni, C. E. (1936). Teoria statistica delle classi e calcolo delle probabilità. *Pubblicazioni del R Istituto Superiore di Scienze Economiche e Commerciali di Firenze*, **8**, 1-62.

Bonferroni, C. E. (1927). Teoria e probabilità. In *Annuario del R Istituto Superiore di Scienze Economiche e Commerciali di Bari per L'anno Accademico 1925-1926*, pp. 15-46.

Bonferroni, C. E. (1941). *Elementi di Statistica Generale* Università Bocconi, Milano (First edition 1927-28).

Galambos, J. and Simonelli, I. (1996). *Bonferroni-type Inequalities with Applications.* Springer-Verlag, New York.

Pagni P (1960). Carlo Emilio Bonferroni. *Bollettino dell'Unione Matematica Italiana*, **15**, 570–574.

M. E. Dewey and E. Seneta