Louis BACHELIER
b. 11 March 1870 - d. 28 April 1946

Summary. Bachelier constructed the first mathematical theory of Brownian motion, and obtained numerous results which are both remarkable and unacknowledged, on the trajectories of stochastic processes.

For a contemporary mathematician, Bachelier's story is strikingly 1different. He was initially overlooked and obtained his first permanent university position at the age of 57, achieving fame only 20 years after his death. Bachelier was born in Le Havre, France, into a family immersed in banking and business. His father, from the Bordeaux region, was a wine merchant and acted as Vice-Consul for Venezuela. His maternal grandfather J.B. Fort-Meu, founded a banking company serving the district of Le Havre. Bachelier matriculated at Caen in 1888, but his father's death forced him to interrupt his studies. He took them up again at the age of 22, and was successful in obtaining an undistinguished Bachelor's degree in Paris in October 1895. He then defended his doctoral thesis, entitled "Théorie de la spéculation" and written under the supervision of Henri Poincaré, on 29 March 1900; this was classified as "honorable". The aim of the thesis was "the application of the calculus of probabilities to stockmarket operations", and was followed by several further notes, papers and original contributions on probabilities until 1914. Bachelier was then mobilised, first as a simple private and later as an officer, until the end of the World War 1.

Bachelier was unable to obtain a university post after he was awarded his doctorate, and had to find non-academic work. Nevertheless, he gave some unpaid lectures at the Sorbonne on the calculus of probabilities from 1910 to 1914. After returning from the war, he first completed an assignment for the Ministry of Labour in Alsace-Lorraine, after which he occupied various precarious positions in the universities of Besançon, Dijon and Rennes. He was finally tenured at Besançon in 1927, where his teaching was well appreciated by his students. He retired in 1937 and until his death lived at St. Malo.

Bachelier's thesis contains three different versions of the first mathematical theory of Brownian motion (five years before Einstein). In modern terminology, Brownian motion was characterized as:
a) a process with independent homogeneous increments whose paths are continuous,
b) the continuous time process which is the limit of symmetric random walks,

and
c) the Markov process whose forward Kolmogorov equation is the heat equation.

Bachelier made a detailed study of the sample paths of Brownian motion thirty years before Paul Lévy, using the reflection principle and the strong Markov property. One may interpret this thesis as the fusion of two seemingly very disparate traditions.

The first, which acted as a guideline for Bachelier, was that of French mathematical physics in the tradition of J. Fourier, G. Lamé and obviously Poincaré. Bachelier drew his analogies, such as the evolution of probabilities, from their ideas. The second was the tacit rational models of stockmarket speculators which were used in a less formalised fashion during the second half of the 19th century. It is possible that a work of Jules Regnault in 1863 already contained, in a more literary form, the conceptual setting for the application of probability to stockmarket operations. In particular, this stated that "the standard deviation of a large number of operations, is in direct proportion to the square root of time". From the mathematical viewpoint, Bachelier's entire thesis is essentially correct; however, as the hypotheses were not always precisely stated, several mathematicians starting with Gevrey and Paul Lévy thought it was gravely in error. The newness of the subject at the beginning of the century, and these negative evaluations resulted in Bachelier's work being overlooked, and when read, not understood.

However, not only was his thesis remarkable, but his later researches were even more so, though they were largely ignored despite a certain measure of support from Poincaré. For example, Bachelier's paper of 1906 provides definitions of the classes of stochastic processes which appeared later: processes with independent increments, Markov processes, Ornstein-Uhlenbeck processes. These definitions appear as consequences of a more general theory: that of stochastic differential equations, which Bachelier developed, without all the rigour to which we are now accustomed, using a vocabulary gleaned from games of chance.

Two functions play a key role in his paper: the first, called "espérance relative à une partie" is what we now call the "drift" of the stochastic differential equation, while the second called "fonction d'instabilité relative à une partie" is the coefficient of diffusion of this equation. Bachelier's arguments concentrate on paths: his equations define a motion, and call to mind Langevin (1908) and, later, work of Ito and Lévy.

In addition, Bachelier introduced an interesting theory of "inverse proba-

bility" and of "probability of causes", that is of statistical estimation, which he pursued in a treatise on the calculus of probabilities published in 1912, which is as clear as it is remarquable. Bachelier was subjected to much derision. In a confidential note to the Rector at Besançon dated 26 May 1921, the Director of Higher Education, providing a confidential evaluation, wrote "His situation is certainly precarious. But he owes it to me, despite the contrary advice of most of the mathematicians. He is not a high flier, and his work is rather peculiar. But he has served well during the war, and we had not been sufficiently fair to him. In effect, he has been placed on trial in your Faculty".

The only scholar who truly recognized the depth of Bachelier's work was Kolmogorov, in his great paper which is at the basis of modern stochastic processes theory "Uber die analytischen Methoden in der Wahrscheinlichkeitsrechnung" of 1931. But after 1930, only Kolmogorov was read, and only those parts of Bachelier's work reproduced in the Soviet author's treatises became known. As for the stock market speculators, they did not need Bachelier's results, which were considered too theoretical to be operationally useful. It was only in the years 1960-1970 that the merit of this French mathematician was recognized by probabilists, and later by financiers, as the new rules for trading options became current. In the 1990's Bachelier Seminars have been created, as has also an international Bachelier Society focussed on the mathematics of finance.

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