Agner Krarup ERLANG
b. 1 January 1878 - d. 3 February 1929

Summary. Erlang's work provided the methodological framework of queueing theory for application to telephone traffic and was a precursor to much modern theory of stochastic processes.

Agner Erlang was born in Lønberg (near Tarm), Jutland, Denmark. He was descended on his mother's side from the famous Krarup family which had strong academic and ecclesiastical traditions. His father was the village schoolmaster and parish clerk and Erlang was initially educated at his father's school. Later he taught at his father's school for for two years before passing the entrance examination to the University of Copenhagen in 1896 with distinction, and the award of a scholarship.

At the University of Copenhagen, Erlang's studies were in mathematics, with minors in astronomy, physics and chemistry. He attended the mathematics lectures of H.G. Zeuthen and S.G. Juel and these gave him a lifelong interest in geometrical problems.

After graduation with a MA degree in 1901, Erlang taught in various schools for 7 years. He was an excellent teacher, but not a particularly social person. During this time he kept up his studies in mathematics and was a member of the Danish Mathematical Association.

It was through meetings of the Mathematical Association that Erlang made contact with the mathematician J.L.W.V. Jensen (1859-1925) (remembered for Jensen's inequality), then chief engineer at the Copenhagen Telephone Company. Jensen introduced Erlang to F. Johanssen, then managing director of the Company, who had recently introduced probabilistic methods into telephony, and Erlang was recruited in 1908. A new physico-technical laboratory was established with Erlang as its head.

Erlang quickly established a conceptual and methodological framework of queueing theory for application to telephone traffic which can be regarded as a precursor of much modern theory of stochastic processes. In 1909 he published his first major work in which he showed that the number of calls during an arbitrary time interval, assuming calls originate at random, follows a Poisson law, and that the intervals between calls were then exponentially distribured. This simple, but physically realistic formulation has provided the benchmark for the subject.

In 1917 Erlang published his most important paper. For an exchange

with R channels, a Poisson stream of incoming calls, and exponentially distributed holding times, he calculated the waiting time distribution and the call loss probability. These formulae are now basic in telephone practice. He introduced the concept of "statistical equilibrium", essentially the modern ergodic hypothesis, which allows the interchange of time and space averages. He also introduced the method of "successive stages", now usually called phases, where lifetimes are divided into fictitious stages, the time spent in each having an exponential distribution. In queueing theory the term "Erlang distribution" is typically used for a sum of independent and identically distributed exponential random variables.

Erlang's writing style was brief and elegant and sometimes proofs were omitted. A.E. Vaulot in France and T.C. Fry in the USA, both important contributors to the subject, studied Danish in order to be able to read Erlang's papers in the original language.

A great deal of attention has subsequently been devoted to the extension and modification of the formulae of Erlang, and to the investigation of their validity. Among the earliest major contributors to this were C. Palm and F. Pollaczek (q.v.).

Erlang was concerned with practical procedures as well as with the theory. For example, he systematized the dealing with stray currents which damaged the lead sheaths of telephone cables. Initially he had no laboratory staff to assist him with the measurement of stray currents. "He could be seen frequently in the streets of Copenhagen followed by a workman carrying a ladder, which was used for the purpose of climbing down into manholes" (Brockmeyer, Halstrom and Jensen, p. 17).

An incidental interest of Erlang's was the extinction of family names, as the Krarup family name of his mother was shortly to become extinct. In 1929 he raised the mathematical problem, essentially as it had been earlier and independently posed by F. Galton (q.v.) in 1873, but had been already independently solved by I.J. Bienaymé (q.v.) in 1845. Erlang provided a partial solution which was completed by the Danish actuary J.F. Steffensen in 1930.

Erlang never married and he devoted himself to his work and studies. He was known as a good and charitable man. He worked for the Copenhagen Telephone Company for nearly 20 years, never having time off for illness until he went to hospital just prior to his death.

The "Erlang" has been used in Scandinavian countries to denote the unit of telephone traffic from the beginning of 1944 and general international

usage followed in 1946.

References

[1] Brockmeyer, E., Halstrøm and A. Jensen (1948). The Life and Works of A.K. Erlang. *Transactions of the Danish Academy of Technical Sciences*, No. 2 (which contains a comprehensive biography, a complete bibliography and Erlang's principal works in English translation).

[2] Syski, R. (1960). *Introduction to Congestion Theory in Telephone Systems*, Oliver and Boyd, Edinburgh (which describes much of Erlang's research in a modern setting).

C.C. Heyde