Mikhailo Pylypovych KRAVCHUK (or KRAWTCHOUK)
b. 27 September 1892 (o.s.) - d. 9 March 1942

Summary. Kravchuk was a Professor of Mathematical Statistics in Kyiv, Ukraine. He discovered the "Krawtchouk" polynomials, and made penetrating contributions to various branches of mathematics before his arrest and death.

Kravchuk ("Krawtchouk" is a transliteration he used when writing in French) was born in the village of Chovnytsia, in the Volyn (Volhynia) region of Ukraine.

After secondary studies at the gymnasium at Lutsk, Kravchuk entered the Physico-Mathematical Faculty at Kyiv (Kiev) University, where in 1916 the algebraist D.O. Gravè described him as one of his most capable students. After a turbulent period on account of World War 1, the revolution and civil war, Kravchuk worked as professor in charge of mathematics and variability statistics in the Kyiv Agricultural Institute (1921-1929), in whose *Zapysky* (*Memoirs*) the Krawtchouk polynomials, a system of polynomials orthonormal with respect to the binomial distribution, make their first appearance in 1929.

He worked simultaneously at a number of other institutions: his first statistical publication appears in 1925 in the *Zapysky* of the Veterinary-Zootechnical Institute; and at the 1928 Bologna International Congress of Mathematicians he is listed as representing Ukraine from the National Institute of Education. In this connection especially he fostered the use of the Ukrainian language in Ukrainian mathematics, was involved in writing school texts, and in the compilation of a Ukrainian mathematical dictionary. The majority of his publications in mathematical statistics, listed and described by Movchan (1972), are in Ukrainian. An abbreviated version of the contents of the 1929 Ukrainian-language paper, without proofs, together with a summary of some additional material to appear in 1931 appeared in French (*C.R. Acad. Sci., Paris*, **189** (1929) 620-622). It is through this version and the definitive treatment of it in Szegö (1939) that Kravchuk's name in its French form was attached to the orthonormal polynomial system. This system, with the aftermath of its creation including the work of Kravchuk's students and associates (O.S. Smohorshewsky, S.M. Kulik, O.K. Lebedintseva) and overseas work, are excellently treated in English in Prizva (1992). This survey also contains a quite detailed description of the contents of a major paper in mathematical statistics of 1931 published after Kravchuk's election to the Ukrainian Academy on 29 June 1929, which has Ukrainian title "Orthogonal polynomials associated with sampling with and without replacement" and French title "Sur les polynômes orthogonaux liés avec les schémas de Bernoulli et de C. Pearson", there being a long French summary of some 9 pages. We return to it below.

Kravchuk was a mathematician of enormous breadth (mathematical statistics was only one of the areas in which he made significant contributions) and energy (some 180 publications in the period 1914-1938 in addition to heavy commitments in the various scientific insitutions). In addition to his work in algebra under the influence of Gravè, he passed on to functions of a real variable and other areas of analysis, differential and integral equations, and geometry as well as mathematical statistics and probability. In the period 1934-1938 he headed the Department of Mathematical Statistics of the Institute of Mathematics of the Ukrainian Academy of Science. His work in mathematical statistics relates to the theory of correlation and regression, the bivariate normal, the method of moments in statistics, and orthogonal polynomial systems. Some of this was influenced by the work of the English Biometric School headed by Karl Pearson (q.v.), as was the early work of Slutsky (q.v.), also in Kyiv in the early years of his career.

A theme running through much of Kravchuk's work, and not only in mathematical statistics, is the method of moments. His statistical direction was initially driven by the work of Chebyshev (q.v.) on interpolation and the method of moments, which in turn derived partly from Chebyshev's contact with the works of Bienaymé (q.v.). In the *Zapysky* version of the 1929 paper, all the substantial papers of Chebyshev on interpolation are mentioned. Kravchuk, however, uses a more modern matrix approach in contrast to Chebyshev's continued fractions. Chebyshev's work was dominant in probability and statistics in the old czarist Russian empire, and this influence of the St. Petersburg School is natural on a strong mathematician such as Kravchuk. The increasing interest in statistics itself surely derived from Kravchuk's work in institutions such as those whose *Zapysky* we have mentioned, where the need for application of statistical methods was transparent.

Kravchuk viewed his polynomial system as a generalization of both the Hermite polynomials (orthonormal with respect to the standard normal) and the Charlier polynomials (orthonormal with respect to the Poisson), inasmuch as both could be obtained in the limit from his. His motivation for studying them was the fact that Chebyshev's polynomials (viewed as orthonormal with respect to the uniform distribution on $0, 1, \cdots, n-1$) were a discrete analogue of the Legendre polynomials, which are obtainable in the limit. A little-noticed feature of Kravchuk's work is the expansion of a binomial probability value in terms of Krawtchouk polynomials orthonormal with respect to another binomial distribution. The germ of the idea is the analogous expansion of densities in terms of Hermite polynomials in Chebyshev's last probability paper (1887); and Kravchuk's expansion also produces the Charlier expansion of a binomial probability in the limit. More importantly, he gives a bound on the error of partial expansion, a very Chebyshevian idea. More importantly still, in his 1931 paper, he produces a polynomial system orthonormal with respect to the hypergeometric distribution. Subsequently this system, overlooked at the time outside Ukraine, even though it was described by Smohorshewsky in the very same French journal (*C.R. Acad. Sci., Paris*, **200** (1935) 801-803) has come to be credited to later work of W. Hahn. Inasmuch as the binomial distribution can be considered as a limit of the hypergeometric, this system generalizes the Krawtchouk polynomials.

Kravchuk was arrested on the 21 February 1938 on a number of false charges, primarily Ukrainian bourgeois nationalism and encouraging foreign espionage because of his contacts with mathematicians outside the Soviet Union, particularly with Ukrainians in Western Ukraine (specifically Levitsky and Chaikovsky, in L'viv), then a part of Poland. He died near Magadan in Siberia. Plaques in his memory have now been erected in Lutsk and in Kyiv, and a small museum established in part of the elementary school in Chovnytsia. An important and well-organized conference dedicated to his memory was held in Kyiv and Lutsk on the 50th anniversary of his death and there was a special number of the *Ukrainian Mathematical Journal* **44**, No.7, 1992. There is some biographical material about Kravchuk in Ukrainian, but little in English or Russian. For a number of years after his arrest he was officially a non-person in the Soviet Union. A general summary of his work, in English, is given by Parasyuk and Virchenko (1992). A brief sketch in English of his life and probabilistic work is given in Seneta (1993).

References

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