Harald CRAMÉR
b. 25 September 1893 - d. 5 October 1985

Summary. Harald Cramér, mathematical master craftsman, contributed pathbreaking research in probability, statistics, and insurance mathematics, and to the illumination of statistics as a coherent mathematical discipline.

Harald Cramér was born in Stockholm, Sweden on September 25, 1893 and spent virtually his whole life there. He entered the University of Stockholm in 1912, studying Chemistry and Mathematics. He made several scientific contributions in Chemistry with H. v. Euler before turning single mindedly to mathematics, becoming a student of the influential Swedish mathematician Mittag-Leffler and studying with the eminent Hungarian mathematician Marcel Riesz, then visiting Mittag-Leffler's Institute. He wrote his doctoral thesis on Dirichlet series, graduating in 1917, and published approximately 20 papers in analytic number theory over the next seven years. This research stood him in good stead later, since in it he became familiar with Fourier integrals of a type closely related to those which were central to his subsequent fundamental probabilistic contributions.

Insurance Mathematician.

As reported in [4], a schism developed between Cramér and the powerful but evidently difficult Mittag-Leffler, who then strongly threatened Cramér's career prospects in pure mathematics in Scandinavia. As Cramér describes it, insurance companies provided a natural alternative employment avenue for young Swedish mathematicians and it was on such a career that he embarked in 1918 (also the year of his marriage). The change of mathematical focus turned out to be very fortunate in Cramér's view, since it introduced him to both insurance mathematics and associated probabilistic developments – leading to his later wide ranging probabilistic interests and contributions.

The insurance mathematics background of the time and Cramér's contributions to the field are well summarized in [4]. He quickly became one of the leading contributors to the basic theory of insurance risks both through his own work and through his influence on colleagues and students. These contributions of course mathematically describe fundamental mechanisms underlying insurance. But as noted in [4], he made even more immediately practical contributions to insurance such as a treatment of loadings in mortality assumptions. He provided a precise discussion of the bias necessary in

premium estimation and developed the 'zero point method' which was used for many years in setting insurance rates.

Most of Cramér's works in insurance mathematics were published in the 1920-1935 period. However his interest in the field was lifelong and he continued his contributions (both technical and educational) well beyond that time even while doing pioneering work in other areas. In particular his classic volume *Collective Risk Theory* was published in 1955 and revised in 1968.

Probabilist.

During the 1920's Cramér was led naturally to an interest in probability through his work in insurance risk. He refers to this period as a "decade of preparation" for the explosive development of probability theory which was to follow, but it nevertheless contained important contributions of his own, as well as the blossoming developments by e.g.. Kolmogorov, Wiener, Lévy. Cramér had already begun using characteristic functions in an extensive study of central limit theory, and motivated by insurance applications he investigated the magnitude of the error in replacing a random sum by its normal limit. He gave an early version of the bound later given final form by his student Esseen in his 1944 thesis, the famous "Berry-Esseen Bound", and detailed Edgeworth (q.v.) expansions for the error.

In 1929 Cramér was appointed to a new chair in "Actuarial Mathematics and Mathematical Statistics" at Stockholm University, set up at the instigation of the Swedish insurance companies. The institute became very active under his leadership and many of the now celebrated results of Cramér himself, and his students and colleagues emanated from it in the ensuing 20 years. The decade of the 30's (described by Cramér as the "Heroic Period") saw dramatic developments in central limit and related theory, involving the Stockholm group (including Feller at that time) as well as that of Khintchine, Wiener, Lévy and others.

Cramér became deeply involved in the developing early theory of stationary processes. In particular he generalized the (1934) covariance representations of Khintchine to a vector context, and the 1938 thesis of his then student H. Wold contained the renowned decomposition of a stationary sequence into its deterministic and purely non-deterministic components.

The years of the second world war caused professional isolation but Cramér maintained a remarkable level of activity at the Stockholm Institute. For example the (1940) thesis by O. Lundberg (q.v.) on stochastic models provided an actuarial basis for non-life insurance which was widely used. He made the

most of the few available opportunities for international contacts. One of these was a primarily Scandinavian conference he organized in Stockholm in 1941 at which he presented the spectral representation of a stationary process (surely one of his most remarkable and exciting results – also obtained independently by M. Loève in the war years).

In the postwar 1940's Cramér's Institute continued its strong stochastic process activity. This included fundamental work of Karhunen on stationary processes, the pathbreaking thesis of Cramér's then student Grenander involving inference for stochastic processes and the work of Cramér himself on general spectral representations for classes of nonstationary processes.

Statistician.

"Probability" is surely the life blood of "statistics" and any dividing line is impossible to position. This is especially true of Cramér's work, much of which may be regarded as "Mathematical Methods of Statistics", as indicated by the title to his most celebrated volume. This enormously influential work was written as a way of making good use of the period of enforced isolation caused by the second world war was described by Cramér as his "Calling card to the postwar scientific world" ([4]). It provided a wonderfully timely and lucid account of a hitherto hodge podge of often mysterious statistical procedures, now organized as a coherent mathematical discipline. It does of course contain many of its author's own contributions, including publication of the celebrated 'Cramér–Rao' bound discovered independently by Rao (1945) *Mathematical Methods* has remarkably few blemishes (one being the constructive definition of Borel sets) and may be perhaps aptly described as "almost uniformly error free". It has had immense influence on generations of statisticians and especially, I would conjecture, in encouraging young (myself included at the time!) mathematicians to enter and find a mathematically satisfying career in statistics and its applications.

One could not describe *Mathematical Methods* as a manual of applied statistics. But it clearly highlights the fact that Cramér was substantially motivated in his mathematical work by the potential for applications. Indeed, as noted in [1], he actually attempted to set up an applied statistics section in the Stockholm group, a plan which did not reach full fruition though a less formal applied activity was instituted.

Administrative, and later years.

In 1950 Cramér was elected President of Stockholm University and in

1958 Chancellor of the entire Swedish university system. This administrative service undoubtedly had a severe impact on his scientific production but by no means halted it, as can be seen from his publications of the period. On his retirement from the Chancellorship in 1961 he again became very active in research. He made several extended visits to the U.S. to participate in research activities at the Research Triangle Institute (RTI) at the invitation of Gertrude Cox (q.v.), and to give lectures.

I first met Harald Cramér at RTI as we worked together there on a project involving the reliability of spacecraft guidance systems, under a contract with NASA. This sparked his interest in level-crossing and extremal problems for stationary processes in which he made important contributions – on which we wrote a joint book [3]. This again underscored his love for development of mathematics relevant to applications.

Simultaneously Cramér investigated questions of "multiplicity theory" and innovation representations" for vector stochastic processes. He wrote a series of approximately 10 basic papers in this area from 1960, the last being published in 1982, at age 89!

Some personal comments.

I have focussed in this biography on Cramér the scientist, and his pioneering research contributions, but there is much to be learned from other sides to his life and work, dealt with e.g. in [1, 4, 5]. Harald Cramér had few peers as teacher and educator, through his written works and incomparably clear classroom teaching, formal lectures and personal interactions. The direct application of his research work is perhaps clearest in the insurance context. But, as indicated throughout, his fundamental theoretical work substantially arose from the need to provide methods for dealing with real applied problems. Thus he strongly facilitated the use of probability and statistics and their wide ranging impact on society. His enormous personal educational impact is surely now less direct but nevertheless has permanently influenced the field.

Cramér's gentlemanly demeanor, courtesy and concern for others were a counter-example to the thesis that professional greatness must be accompanied by some form of social or character weakness. Finally, no record of the life of Harald Cramér should omit mention of his wife Marta – his lifelong companion from their 1918 marriage until her death in 1973. No doubt Cramér would have made pioneering contributions if he had remained single or had a less successful marriage. But the extraordinarily fine mutual sup-

port and life partnership with his "Beloved Marta", and their strong family relationships gave an extra quality to his – indeed their – professional and human legacy.

Harald Cramér died in his native Stockholm on October 5, 1985, with the conviction he expressed just prior to his death, that his life's work was now complete.

Further reading

Several detailed accounts are available of the life and work of Harald Cramér and his fundamental contributions in [1, 4, 6, 5, 7]. Especially illuminating is Cramér's own description in [2] which gives a fascinating account of this exciting period in probability and statistical science and his role in it. A reader interested in the details of Cramér's contributions and their motivating environment will be rewarded by study of [2] and the other references cited. In particular, [1] contains a comprehensive bibliography of his published works.

References

[1] [1] Blom, G. (1987). Harald Cramér 1893-1985. *Annals of Statistics*, **15**, 1335-1350.

[2] [2] Cramér, H. (1976). Half a century with probability theory: Some personal recollections. *Annals of Probability*, **4**, 509-546.

[3] [3] Cramér, H. & Leadbetter, M.R. (1967). *Stationary and Related Stochastic Processes*. Wiley, New York.

[4] [4] Grenander, U. (1994). A survey of the life and works of Harald Cramér, *Scandinavian Actuarial Journal*, (1995) **1**, 2-5.

[5] [5] Leadbetter, M.R. (1988). Harald Cramér 1893-1985. *International Statistical Review*, **56**, 89-97.

[6] [6] Martin-Löf, A. (ed.) (1994). *Harald Cramér: Collected works*. Vols. I & II, Springer-Verlag, Berlin.

[7] [7] Leadbetter, M.R. (1995). A view of Harald Cramér: working with the master in his later years. *Scandinavian Actuarial Journal*, **1**, 6-11.

M.R. Leadbetter