George BOOLE
b. 2 November 1815 - d. 8 December 1864

Summary. Boole's name in probability theory and statistical inference is preserved by Boole's Inequality, an important tool for dealing with the presence of statistical dependence. In the design of electronic circuitry, Boolean algebra is widely used.

Boole's name is well known in connection with the founding of symbolic logic, and entries on him may be found, primarily in this context, in major reference sources such as the *Encyclopaedia Britannica*, *Encyclopedia of Philosophy, Dictionary of Scientific Biography* as well the well-known *Men of Mathematics* of E.T. Bell. Symbolic logic, simplistically speaking, uses for class-terms, propositions and relations between them, algebraic symbols and relational symbols much in the way that is done in set theory. This permits a mechanistic solution, as in algebra, of logical problems. Commensurately named *Boolean algebra* is now widely used often in connection with binary codes and electronic switching in computer science. Since modern probability theory according to the Kolmogorov axiomatization is based on set theory, it is not in retrospect surprising that Boole had a strong interest in probability theory as an adjunct to formal logic, and was indeeed part of a British tradition of this kind, with his near contemporaries De Morgan (q.v.), Jevons (q.v.), and Venn (q.v.).

Boole's (1854) major study *An Investigation into the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities*, has remained in print. In this British tradition, too, was Lewis Carroll (Charles Lutwidge Dodgson, 1832-1898) whose paradoxes and books in a lighter genre, *Symbolic Logic* and *The Game of Logic*, have continued also to attract interest.

It is worth mentioning that, specifically on the probabilistic side of the tradition, of which De Morgan stands as head, the books of Isaac Todhunter (1820-1884) *A History of the Mathematical Theory of Probability from the Time of Pascal to That of Laplace* (1865) and William Allen Whitworth (1840-1905) *Choice and Chance* (1867) have also continued to be in print and in use, a remarkable record of longevity. Boole's own books in their own time did not enjoy popularity; they were said, by a superficial commentator (*Athenaeum*, December 17, 1864) given to the *bon mot* to have:

" ... sought a very limited audience, and we believe, found it."

Boole was born in humble circumstances in the city of Lincoln in England, where his father was a shoemaker, and a mathematical amateur given to the construction and adaptation of optical instruments, a taste which was passed onto George, who revered him. George's schooling was ordinary, although he showed prodigious talent, and his early ambition ran in the direction of attaining proficiency in classical Greek and Latin. He was essentially self-taught in mathematics from the age of about 17 when he began as teacher in small schools, eventually opening his own day-school for youth of both sexes in Lincoln, then going on to run a school at which he had formerly taught, in Waddington, to support his parents and other members of his family. From Waddington came his earliest mathematical papers with the support and encouragement from 1839 of one of the editors of the newly founded (1837) *Cambridge Mathematical Journal*, Duncan Farquharson Gregory (1813 - 1844). With Gregory's encouragement, in 1844 Boole communicated a paper to the Royal Society of London. This was awarded the Royal Medal in that year as the most important paper communicated to its *Transactions*. In 1849 Boole was rescued from schoolmastering by being selected to fill (even though he had no formal university degree) the office of Professor of Mathematics in the newly formed Queen's College, in Cork, where he was prolific in his research and active in the scientific context of Ireland till his death.

In 1855, Boole married Mary Everest, niece of Colonel Everest after whom Mount Everest was named, an intellectually active woman with whom he had five daughters. He was elected Fellow of the Royal Society in 1857. The Queen's Colleges of Belfast, Galway and Cork were united to form the Queen's University of Ireland at about this time and Boole was appointed one of the public examiners, a position which he filled with eminence. His premature death, at Ballintemple, County Cork, came about as a consequence of lecturing in clothes wet from the rain.

The burden of supporting his young family fell on his wife, Mary Everest Boole, who moved back to London. The youngest daughter, Ethel Lilian eventually married a Polish revolutionary, and as Ethel Lilian Voynich (1864 - 1960) achieved fame in the Communist bloc as author of *The Gadfly*, (in Russian: *Ovod*), whose hero was held to be a model for Communist youth. It appears, paradoxically, that the book was based on her association with Sidney Reilly, English spy and enemy of the Bolsheviks (MacHale, 1985, pp.269-276). Karl Pearson (q.v.) through his interest in socialism and women's emancipation, has a cameo part in her history. Several of George Boole's great grandchildren, from another daughter, attained scientific eminence in Britain and the U.S. in our own time.

In probability theory, and therefore by extension in statistics, Boole's name is attached to *Boole's Inequality*: for events $\{A_i\}, \quad 1 \le i \le n$,

$$P\left(\cup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$$

which can be written equivalently, through use of complementation, as

$$P\left(\cap_{i=1}^{n} B_i\right) \ge 1 - \sum_{i=1}^{n} (1 - P(B_i))$$

where $B_i = \bar{A}_i$. For Boole's various inequalities as they actually occur in his *An Investigation ...*, see Seneta (1992). The strength of the above inequality is that it holds without events being mutually exclusive or independent. Dependence of events defined by successive dependent sample means $\bar{X}_{(n)} = \sum_{i=1}^{n} X_i/n, \quad n \ge 1$, where the X_i's are independent random variables, was the obstacle Francesco Paolo Cantelli (1875-1966) was able to overcome with the aid of Boole's ("forgotten") Inequality, to give in 1917 a proof of the Strong Law of Large Numbers for general random variables, after an attempt in 1909 by Borel (q.v.) for Bernoulli random variables. Boole's Inequality is the first of a sequence of inequalities of increasing complexity ("degree") which have found use in simultaneous statistical inference because they can cope with probabilities of unions of arbitrary dependent events. The second inequality of the sequence is:

$$P\left(\cup_{i=1}^{n} A_i\right) \ge \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j);$$

and the inequalities for $P(\cup_{i=1}^{n} A_i)$ continue in this way, with alternation of inequality sign from one inequality to the next in sequence. The sequence is known as the Bonferroni Inequalities, after Carlo Emilio Bonferroni (q.v.) who explicitly, in 1936, attributes the first inequality to Boole. However usage of the first inequality in statistical inference is generally mistitled the Bonferroni adjustment, presumably through the more musical quality of the name. Nothing is ever named after its discoverer: Stigler's Law of Eponymy.

References

[1] Boole, G. (1952). *Studies in Logic and Probability.* (Ed. Rush Rhees) Watts and Co., London.

[2] Hailperin, T. (1986). *Boole's Logic and Probability.* (2nd edn.) North Holland, Amsterdam.

[3] Harley, R. (1866). George Boole, F.R.S. *British Quarterly Review*, July, 141-181. [Also as pp. 425-472 of Boole (1952)].

[4] Heath, P. (1967). Boole, George (1815-1864) *Encyclopedia of Philosophy* (Paul Edwards, Ed.) Macmillan Free Press, London, pp.346-347.

[5] Kneale, W. (1948). Boole and the Revival of Logic. *Mind*, **57**, 149-175.

[6] MacHale, D. (1985). *George Boole. His Life and Work*. Boole Press, Dublin.

[7] Seneta, E. (1992). On the History of the Strong Law of Large Numbers and Boole's Inequality. *Historia Mathematica* **19**, 24-39.

Peter Heath and Eugene Seneta