Carl Friedrich GAUSS
b. 30 April 1777 - d. 23 February 1855

Summary. Gauss shaped the treatment of observations into a practical tool. Various principles which he advocated became an integral part of statistics and his theory of errors remained a major focus of probability theory up to the 1930s.

Gauss was born on 30 April, 1777 in Brunswick, Germany, into a humble family and attended a squalid school. At the age of ten, he became friendly with Martin Bartels, later a teacher of Lobachevsky. Bartels, an assistant schoolmaster in Gauss's school, studied mathematics together with Gauss and introduced him to influential friends. From 1792 to 1806 Gauss was financially supported by the Duke of Brunswick. He was thus able to graduate from college (1796) and Göttingen University (1798). He then returned to Brunswick and earned his doctorate from Helmstedt University (1799). Only in 1807 did Gauss become director of the Göttingen astronomical observatory (completed in 1816), and his further life was invariably connected with that observatory (and the university of the same city). Gauss was twice married and had several children, but none became scientists. He died in Göttingen on 23 February, 1855.

Gauss is regarded as one of the greatest mathematicians of all time. He deeply influenced the development of many branches of mathematics (e.g., algebra, differential geometry) and initiated the theory of numbers; he was an illustrious astronomer and geodesist, and together with Weber he essentially contributed to the study of terrestrial magnetism. Gauss's importance for developing the mathematical foundation for the theory of relativity was "overwhelming" (Einstein, quoted by Dunnington on p.349 without an exact reference). His command of ancient languages was exceptional, and, until he discovered the possibility of constructing a regular 17-gon with a straightedge and compasses, he had remained undecided whether to pursue mathematics or philology. He also possessed an admirable style in his mother tongue.

Gauss was vey slow in making known his findings, many of which were published posthumously. He kept silent about his studies of the "anti-Euclidean" geometry, as he called it, although he (successfully) nominated Lobachevsky for Corresponding Membership of the Göttingen Royal Scientific Society. He was showered with honours from leading academies. In 1849, he became honorary citizen of Brunswick and Göttingen. He had no peers in science and remained isolated, partly because of his own disposition. He was reluctant to refer to other authors and did not befriend younger scholars (e.g., Jacobi, Dirichlet). Gauss attached great importance to such problems as the relation of man to God, but thought that they were insoluble. He believed in enlightened monarchy; however, in a letter, he wrote about the golden age to be expected in Hungary after the 1848 revolution.

As an astronomer, Gauss is best known for determining the orbits of the first minor planets from a scarce number of their observations and calculating their perturbations, and for contributing to practical astronomy. He and Bessel independently originated a new stage in experimental science by introducing thorough examination of instruments. Gauss also detected the main systematic errors of angle measurements in geodesy and outlined means for eliminating their influence. For about eight years he directly participated in triangulating Hannover. After 1828, he continued to supervise the work, which ended in 1844, and he alone performed all the calculations. His celebrated investigation of curved surfaces and study of conformal mapping were inspired by geodesy.

Gauss solved several interesting probability-theoretic problems. In 1841, Weber described Gauss's opinion that, in its applications, probability should be supplemented by "other knowledge" and that the theory offers clues for life insurance and for determining the necessary numbers of jurors and witnesses. Gauss also studied the laws of infant mortality and for several years directed the widows' fund at Göttingen University.

The treatment of observations occupied Gauss at least from 1794 or 1795. He decided that redundant systems of physically independent linear equations should be solved according to the principle of least squares. He applied it in his astronomical and geodetic work and recommended it to his friends. In 1809 he published its justification. Issuing from a postulate that the arithmetic mean of direct measurements of a constant should be assumed as its value, and making use of the principle of maximum likelihood, he arrived at the normal distribution of observational errors as their only possible even and unimodal law. He also supposed a uniform prior distribution of errors; this, hovever, was already implied by his postulate. He substantiated maximum likelihood by the principle of inverse probability. Gauss claimed to be the inventor of least squares although Legendre (q.v.) had introduced it publicly (without justification) in 1805. For him, priority always meant being first to discover.

In 1823 (supplemented in 1828) Gauss put foward a new substantiation of least squares pointing out that an integral measure of loss (and, more definitely, the principle of minimum variance) was preferable to maximum likelihood, and abandoning both his postulate and the uniqueness of the law of error. (The normal law still holds, more or less, on the strength of the central limit theorem. In 1888, Bertrand (q.v.) nastily remarked that for small values of $|x|$ any even law $f(x) = a^2 - b^2x^2 \approx \alpha^2 \exp(-\beta^2 x^2)$.) Also in 1823, Gauss offered, for unimodal distributions, an inequality of the Bienaymé - Chebyshev type, and another one, for the fourth moment of errors, as well as the distribution-free formula for the empirical variance, m^2, and for its own variance, Dm^2. Owing to an elementary error, his Dm^2 was wrong; first Helmert (1904), then Kolmogorov et al (1947) corrected it. Gauss set high store by the formula for m^2 which provided an unbiased estimate of σ^2; however, in geodetic practice precision is characterized by its biased estimator, m, and Helmert thought that only relative unbiasedness was important.

Gauss also estimated the precision of the estimators of the unknowns of his initial linear system, and of linear functions of these. Partly owing to his apt notation, his method of solving normal equations by eliminating the unknowns one by one became standard. He also applied iterative processes (as described by Dedekind), and he introduced recursive least squares which mathematicians did not notice until recently. In 1816, he proved that, for normally distributed errors e_i, the measure of precision $h = 1/\sqrt{2}\sigma$ was best estimated by $(e_1^2 + e_2^2 + \ldots + e_n^2)$, rather than by $(|e_1|^k + |e_2|^k + \ldots + |e_n|^k)$, for any other integer k.

Gauss's contribution to the treatment of observations somewhat extended by Helmert defined the state of the classical theory of errors. It seemed perfect, and geodesists hardly paid attention to statistics, whereas statisticians hardly studied Gauss and thus missed the opportunity to develop analysis of variance and regression with less effort. This situation did not begin changing until well into the 20th Century.

References

[1] C.F. Gauss:

[2] 1973-1981 *Werke*, Bde 1-12. Hildesheim. This is a reprint of the previous

edition (1870-1930) which includes contributions (with separate paging) on the work of Gauss in geodesy (A. Galle) and in astronomy (M. Brendel), both in Bd. 11, Tl. 2; and on Gauss as a calculator (Ph. Maennchen, in Bd.10, T1. 2).

[3] 1975-1987 *Werke*, Ergaenzungsreihe, Bde 1-5. Hildesheim. Reprints of previously published correspondence of Gauss.

[4] 1964 *Abhandlungen zur Methode der kleinsten Quadrate*. Eds, A. Boersch & P. Simon. Wuerzburg. Reprint of the edition of 1887. Contains translations from Latin/reprints of all Gauss's works on the theory of errors (mainly 1809; 1811; 1816; 1823; 1828, as well as their abstracts compiled by Gauss himself). Russian transl. of same material with foreword by G.V. Bagratuni; Gauss (1957). French transl. of essentially the same material, Paris 1855.

[5] 1957 - 1958 *Izbrannye geodezicheskie sochinenia* (Sel. geod. works), vols.1-2. Moscow.

[6] 1995 *Theory of the combination of observations least subject to error*. Latin and English. Transl. with Afterword by G.W. Stewart. Philadelphia.

[7] Other authors:

[8] Dedekind, R. (1931). Gauss in seiner Vorlesung ueber die Methode der kleinsten Quadrate (1901). *Ges. math. Werke*, Bd.2, Braunschweig, pp. 293-306.

[9] Dunnington, G.W. (1955). *C.F. Gauss, Titan of Science*. New York.

[10] Forbes, E.G. (1978). The astronomical work of C.F. Gauss. *Historia Mathematica*, **5**, 167-181.

[11] Hald, A. (1998). *A History of Mathematical Statistics from 1750 to 1930*, Wiley, New York. See also the contributions of Helmert (1872), Plackett (1972), Seal (1967) and Sprott (1978) referenced therein.

[12] Reichardt, H., Editor (1957). *C.F. Gauss. Gedenkband*. Leipzig. Includes O. Volk, Astronomie und Geodaesie bei C.F. Gauss, pp. 205-229. The book was reprinted (Berlin, 1960) as *C.F. Gauss. Leben und Werk*.

[13] Sartorius von Waltershausen (1965). *Gauss zum Gedaechtniss.* Wiesbaden. First Published in 1856.

[14] Sheynin, O.B. (1979; 1993; 1994). Three papers from *Archive for History of Exact Science*. Among the references in the first of these see Loewy (1906), May (1972), Sofonea (1955) and Subbotin (1956).

[15] Sheynin, O.B. (1996). *The history of the theory of errors*, this being Deutsche Hochschulschriften No.1118. Egelsbach a.o.

O.B. Sheynin