Rogerius Josephus BOSCOVICH
b. 18 May 1711 - d. 13 February 1787

Summary. Rogerius Josephus Boscovich was a Jesuit priest from Dubrovnik, Dalmatia, who developed in 1760 a simple geometrical method of fitting a straight line to a set of observations on two variables using (constrained) least sum of absolute deviations. This procedure was subsequently popularised in an algebraic form by Laplace.

Rogerius Josephus Boscovich (Italian: Ruggiero Giuseppe Boscovich; Croatian: Rugjer (or Rudjer) Josip Bošković) was born in Ragusa (now Dubrovnik, Croatia), the sixth son of a merchant's agent. He was educated at the (Jesuit) *Collegium Ragusinum* in Ragusa and at the (Jesuit) *Collegium Romanum* in Rome (Italy), but long before he had completed his training as a Jesuit priest he discovered his vocation as a teacher of mathematics. Indeed, in 1740 he succeeded his own teacher as professor of mathematics at the *Collegium Romanum*. In the years following 1743 Boscovich was employed by Pope Benedict XIV as an adviser on a variety of mathematical problems. In particular, in 1750-1752 Boscovich and the English Jesuit Christopher Maire were commissioned to measure an arc of the meridian in the vicinity of Rome and to prepare a new geographical map of the Papal States. A detailed account of this geodetic survey was published by Maire and Boscovich in 1755 and a summary by Boscovich alone in 1757.

The Seven Years War of 1756-1763 was fought between Britain, Hanover and Prussia on one side and Austria and France on the other, and resulted in the loss of the French territories of Québec and Maine in North America. This war brought Boscovich's skills as a diplomat into play. In 1758 he was sent to Vienna (Austria) and in 1760 to Paris (France) and London (England). During his visit to London, Boscovich finalised his notes for the second volume of the poem in Latin hexameters by his fellow Ragusan Benedikt Stay. He also met many leading members of the Royal Society of London and took the opportunity of proposing a geodetic expedition to North America. Later, in 1767, Boscovich was invited to participate in the 1769 astronomical expedition to California, but his invitation was withdrawn when the organisers discovered that Jesuits were strictly excluded from Spanish territory in the Americas.

The Jesuit Order was suppressed throughout the world in 1773. however Boscovich managed to secure a pension from King Louis XV of France which lasted until his death in Milan (Italy) in 1787.

Boscovich made significant contributions to a wide range of scientific disciplines, his principal contribution being the first modern treatise on the atomic theory of matter. For detailed discussions of his scientific work, see Bursill-Hall (1994) and Whyte (1961). Here we shall be concerned with Boscovich's development of the first *objective* procedure for fitting a linear relationship to a set of observations. [The earlier procedure of Tobias Mayer is not objective as the classification of the observations into groups clearly depends on the criterion employed in their classification.]

The fundamental mathematical problem which Boscovich addressed in his section of Maire and Boscovich (1755, pp. 497-503; 1770, pp. 479-484), in Boscovich (1757, pp. 391-392; 1961, pp. 88-93) and in his notes to Stay (1760, pp. 420-425; 1770, pp. 501-510) was that of determining the values of the coefficients a and b which best fit n equations of the form

$$y_i = a + bx_i \quad i = 1, 2, \ldots, n$$

The problem explicitly addressed by Boscovich in his 1755 and 1757 studies concerned his attempt to reconcile the ten distinct values $b_{ij} = (y_i - y_j)/(x_i - x_j)$ which he had obtained for the slope coefficient by solving the $n = 5$ equations two at a time. In 1755 Boscovich had no firm idea of what to do with these ten values. He first took an unweighted average of all ten values, and then an average of the eight values which remained after he had deleted the two with smallest denominators. However, in 1757 he formulated the principle that the values of a and b should be chosen in such a way that the errors

$$e_i = y_i - a - bx_i \quad i = 1, 2, \ldots, n$$

should sum to zero and have minimum absolute sum.

From the adding-up condition, $\sum e_i = 0$, we find that a is related to b by $a = \bar{y} - b\bar{x}$ where $\bar{x} = \sum x_i/n$ and $\bar{y} = \sum y_i/n$. Substituting this expression into the optimality condition, we find that we have to minimise $\sum |(y_i - \bar{y}) - b(x_i - \bar{x})|$, that is, we have to find the weighted median of the ratios $b_i^* = (y_i - \bar{y})/(x_i - \bar{x}) \;\; i = 1, 2, \ldots, n$.

Boscovich did not explain how this problem could be solved in his 1757 summary but merely gave the results of the calculations. However, in his commentary to Stay's poem he showed how the ratios may be calculated and arranged in decreasing order by means of a geometrical procedure. This geometrical procedure takes the form of a cartesian diagram with axes $x = 0$ and $y = 0$ in which the n points (x_i, y_i) $i = 1, 2, \ldots, n$ have been plotted together with a movable line. This movable line starts in the vertical position $x = \bar{x}$ and rotates in a clockwise direction about the centroid $(\bar{x}, \bar{y})$. As it moves, the line passes through each of the n points in turn and identifies the successive ratios in decreasing order. The weighted median of these ratios, and hence the minimal value of the absolute error function $\sum |e_i|$, is identified by evaluating the cumulative sum of the weights $|x_i - \bar{x}|$ $i = 1, 2, \ldots, n$ in the order determined by the moving line, and choosing the point for which this cumulative sum first equals or exceeds one-half of the sum of all n such terms.

Boscovich's use of geometrical terminology in this context was disapprobated by Laplace (q.v.) who translated his procedure into an algebraic format in 1793. Nevertheless, it should be noted that Boscovich probably obtained the ideas embodied in his solution procedure from a geometrical diagram of this type. the restricted variant of this procedure discussed by Boscovich and Laplace was subsequently generalised to any number of explanatory variables (with or without the adding-up constraint) by Gauss (q.v.) in 1809. However, Gauss seems to have made no further use of the procedure after that date. See Farebrother (1987) for an account of the later history of this L_1 procedure (subsequently renamed the method of situation by Laplace) and the closely related minimax absolute error or L_∞ procedure. More recently, the L_1 procedure has served as the basis of a wide class of robust fitting procedures.

Boscovich has been severely criticised by later writers for choosing to publish some of his original scientific work in the notes to Stay's poem. Even if we allow that he could write in Latin with ease, it seems inconceivable that he should have chosen to devote so much of his spare time to writing a detailed commentary on a poem in Latin hexameters. However, this is precisely what he did choose to do. Further, it seems reasonable to conjecture that Boscovich would have thought that this manner of spending his leisure time compared very favourably with that employed by his wealthier contemporaries (not to mention present-day statisticians).

References

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