Christiaan HUYGENS
b. 14 April 1629 - d. 8 July 1695

Summary. Trained to become a diplomat, a career which did not eventuate due to political circumstances, Christiaan Huygens turned to science and mathematics. While in Paris he heard of the Pascal's and Fermat's attempts to solve gaming problems. Back in Holland he conceived his calculus of expectations which considerably influenced the following generation of probabilists.

Christiaan Huygens was born in the Hague on April 14, 1629, the son of the diplomat, writer, and poet Constantijn Huygens whose Dutch and Latin verse gained him a lasting place in the history of Dutch literature. Because of its services to the house of Orange in two generations the Huygens family had risen to high social rank. Christiaan was educated by private tutors and his father before he went to Leiden in 1645 to study law and mathematics with the younger Frans van Schooten. He studied classical Greek mathematics and the new methods of Viète, Descartes, and Fermat (q.v.).

In March 1647 Christiaan Huygens matriculated at the Collegium Auriacum (Orange College) in Breda, again to study law. He also privately continued his studies of mathematics. After his return to the Hague in August 1649 he went to Denmark as a member of a diplomatic mission in the fall of 1649. However, after the death of the stadholder William II in 1650, Huygens had no chance to enter the diplomatic service for which he was prepared by his education.

The 17 years between the end of his studies in Breda and his departure to Paris in April 1666 which were spent in the Hague but with short visits to Paris and London were the most fertile of Huygens' career. He worked from 1650 to 1666 supported by an allowance supplied by his father, as a gentleman scientist, on problems of mathematics, mechanics, astronomy, the construction of pendulum clocks and of optical instruments, the lenses of which he ground himself. The discovery of the ring of Saturn in 1655/56 and the invention of the pendulum clock in 1656 made him famous and by the mid 60's he was considered the leading mathematician and natural philosopher of his time.

The following 14 years he spent in Paris at the Academy of Science. He continued his work in mathematics, mechanics, astronomy and technical problems and added to his interests fundamental questions concerning the cause of gravity and the nature of light.

The time in Paris was interrupted by several stays in the Hague where he tried to cure an illness which began in 1670. In 1672 the Netherlands were attacked by Louis XIV. William III, prince of Orange, was appointed Stadholder and Christiaan Huygens' father together with one of his sons played a considerable role in the defence against the French. One of the most urgent problems the Netherlands had to solve in this situation was to recruit soldiers for the military defence. The Radspensionaris of Holland, Jan de Witt, had worked out a financial arrangement, the Waerdye, according to which annuities on lives appeared as the most favourable means for the state to raise funds for this purpose. De Witt's expertise was based on the same principle as Huygens work on games of chance. Christiaan Huygens was informed of these activities but he seemingly felt no tensions concerning his loyalty. He not only remained in Paris but also dedicated his book on the mathematical theory of the pendulum clock, in 1673, to the French king.

When he again returned to the Hague in September 1681 because of his illness he decided not to return to Paris where the situation for protestants even from foreign countries had deteriorated. In the years until his death on July 8, 1695 he finished a series of works amongst which his *Traité de la lumière* published in 1690 and containing his theory of light and ideas about gravity, is perhaps the most famous.

Huygens' encounter with the world of stochastics took place quite early in his career. During his first stay in France in 1655 he had heard about letters exchanged in 1654 between Pascal (q.v.) and Fermat in which the two had discussed problems concerning games of chance. At that time Huygens neither met with Pascal or Fermat nor could he gather details about the methods used in the solution of these gambling problems. However, shortly after his return from France Huygens worked out a method to solve the problems based on his understanding of "expectation". From a conceptual point of view Huygens' "expectation" is different from "expectation" in later probability theory but both concepts yield the same values in the cases treated by Huygens. Huygens did not use the word probability in his solutions of chance problems. In his own understanding the method he applied for these solutions served only the purpose to demonstrate the power of the new algebra created by Viète and Descartes. The truth of Viète's statement that the new algebra leaves no sensible problem unsolved could now be demonstrated by Huygens' success to apply algebra to the realm of chance which hitherto seemed inaccessible for mathematics. When Huygens informed his former teacher Frans van Schooten of his work on games of chance van Schooten offered him space for a publication in his forthcoming book. So Huygens' tract *De ratiociniis in ludo aleae* came out 1657 in Leiden as an appendix to Frans van Schooten's *Exercitationum Mathematicarum Libri Quinque.*

All the problems solved by Pascal, Fermat, and Huygens can be reduced to two problems, the problem of points and the problem of dice, both of which can be traced back at least to the late middle ages. The problem of points presupposes that a game is not decided by just one attempt of e.g. throwing a die but by a whole series of single attempts. In general the parties involved agree that the winner of the whole game and by that of all the stakes is the party which first won a certain number of single games. If the whole game cannot be finished for some reason and the parties have to leave before any of them has reached the necessary number of wins, the stakes have to be divided according to the number of wins the respective parties are lacking. Since the whole game can be interrupted at any time, those concerned with the solution of the problem of points began at least in the 14th century to investigate the new distribution of the stakes after each single game. So it became interesting to know how much of the opponent's stake would go to the side of the winner of a single game, be it the first, the second, and so on up to the last game.

The problem of dice was formulated in the mid-17th century as asking how many throws of a die are needed to get at least n aces. Huygens based the solution of these problems on the principle of a just game of chance. Participants engage in a game of chance because they hope for a gain and they know that they have to pay for this hope with the risk of a loss. A presupposition for justness is that the sum of the stakes is equal to the sum of the payoffs of the players, or, in other words, that there is no third party who takes a share of the stakes for his service to organize the game. Whatever game of chance is played, its end, that is to say, who won and who is entitled to the loser's stake is clear by unambigous rules which were valid centuries before Huygens. Huygens generalized the situation by admitting that the winner is entitled to a part of the loser's stake. This part can be less than the whole stake, but, of course, it must be positive. Huygens defined a just game with the postulate, that in gambling the expectation or share that somebody can claim for something is to be estimated as much as that with which, having it, he can arrive at the same expectation or share in a fair game. Huygens' fundamental principle contains the term expectation which is not explained explicitely. The expectation of a player A engaged in a game of chance in a certain situation is identified with his share of the stakes if the game is not played or not continued. If the game is not played or continued with player A who will be replaced by a player B, B has to refund A by an amount equal to the expectation of A in this situation in order to engage in a just game. Chance was considered by Huygens and his predecessors as a self evident term. Chance meant for them an unpredictable and hence uncertain event. In order to subjugate chance to mathematics it was necessary to select a class of these unpredictable events characterized by equipossibility which was considered by Huygens as something elementary and clear. As paradigms for equipossible cases were offered the outcomes of throwing a die, tossing a coin, participation in a lottery, or choosing between two hands hiding different amounts of money. The more complex problems were solved by reduction to equipossible cases. Huygens and his predecessors still lacked a problem which forced them to go beyond this. But a generation later, with Jakob Bernoulli (q.v.), new cases of unpredictability like dying in a certain age were taken into consideration which seemed to be neither equipossible nor reducible to equipossibility. For the successors of Huygens it began to matter that a more skilled and able player would win more frequently than his opponent in a series of games. Because equipossiblity had not become problematic to him frequency did not figure in Huygens' tract of 1657.

The first three propositions in Huygens' tract served for the determination of expectations in concrete situations. The first proposition is that if it is equally easy to obtain the amount a or b the value of the expectation is $(a+b)/2$, while the second proposition deals similarly with three equally likely amounts. Huygens' central proposition 3 says that if the number of cases for gaining a is p, and the number of cases for gaining b is q, then assuming that all cases can happen equally easily, the expectation is $(pa + qb)/(p + q)$. The first three propositions show that there is no need for Huygens to revert to any notion of probability as available at the time in order to explain his understanding of expectation or his way to determine it. Instead Huygens felt a need to demonstrate the first three propositions of his tract in the most rigorous way.

In all three cases Huygens based his proof on a system of mutually symmetrical contracts between players. However, an analysis of the proofs of propositions 2 and 3 shows that they hold only if the very meaning of winning a game, and by that, common sense is given up. Huygen's proofs allow the possibility that the winner of a game between players who staked the same amount could go away with less than any of the losers.

Huygens was very much aware that some would accuse him of support with his tract for the frivolity of gaming, but he hoped that most of his readers would appreciate the utility of his work. Huygens uses the first three propositions to solve 11 problems in propositions 4 to 14. The first six problems deal with different situations of the problem of points presupposing equal chances for the players involved. Huygens' procedure for the solution of the problem of points involved establishing a difference equation for recursive calculation of expectations. Then, from the situation of equal chances Huygens proceeded to problems involving unequal chances for which his paradigm was the throwing of dice. Typical is the ninth problem (proposition 12): "To find how many dice should one take to throw two sixes at the first throw." Huygens added to the 14 propositions of his tract five problems the solutions of which he left to his readers. For the first, third and fifth of these problems which had been posed by Fermat and Pascal he gave the numerical results without the appropriate reasoning. This is the content of Huygens' tract from 1657 covering some 20 pages which came out again in the original Dutch wording in 1660. Because of its enormous impact on the following generation of mathematicians concerned with stochastic problems it has been called the first book on mathematical probability. However,it was neither a book - it was published as a short appendix to a book of Frans van Schooten - nor was it on probability since neither the word nor a concept of probability were used in the tract. Huygens' "theory" of games of chance was the intellectual game of a mathematician.

After 1657, Huygens repeatedly returned to problems of chance. In every case he was induced to these activities by others. He was eager to demonstrate that his method of calculating expectations by recursion sufficed to solve problems for which others used combinatorial methods. None of these later stochastic considerations was published; so his later work, had no impact on the following generations of mathematicians. When John Graunt (q.v.) brought out his *Observations Made upon the Bills of Mortality* in 1662 a copy was sent to Christiaan Huygens. The *Observations* contained a very short life table and this was used by Christiaan to discuss with his brother Lodewijk in a correspondence from 1669 on the basis of his calculus of expectations in games of chance and the lottery model the difference of what is called today the expected and the median lifetime; in addition he approached the problem of joint-life expectations. The letters exchanged between the two brothers in 1669 would have been an important source of inspiration for all who dealt with mortality problems shortly afterwards but remained unknown until their publication in the Oeuvres in 1895. On the other hand Huygens' tract of 1657 was, relative to the small number of active mathematicians at the time, one of the most influential papers in the history of mathematics. In England Huygens' tract appeared in an English translation extended by the combinatorial methods propagated by Pascal in 1692. Jakob Bernoulli who transformed Huygens' concept of expectatation and used this to introduce the classical measure of probability reprinted Huygens' tract together with his annotations in the first book of his *Ars conjectandi* the first book centered around probability as the main concept of a new mathematical theory.

Bibliography

A good account of Huygens' tract and his other activities in stochastics even if with the unjustified assignation to probability theory is given in:
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