Joseph BERTRAND
b. 11 March 1822 - d. 3 April 1900

Summary. Bertrand brought several interesting innovations into the probability calculus, but it is above all for his role as teacher, as publicist, and as critic (indeed as polemicist) that he is known to history.

Joseph Bertrand, who was born and died in Paris, was a prodigy who fulfilled his childhood promise. He was the son of Alexandre Bertrand, a graduate of the Ecole Polytechnique who was an expert on sleepwalking, ecstasy and other extraordinary states of consciousness. Joseph, orphaned at the age of nine, was raised by his uncle, the mathematician J.M.C. Duhamel (1797-1872), who left him free to study as he pleased. Joseph Bertrand held all the French records for precocity at the university; he was allowed to follow lectures at the Ecole Polytechnique at the age of 11, and was awarded his doctorate in the mathematical sciences at sixteen. He was then admitted as the top candidate at the Ecole Polytechnique, having at last reached the age required to compete officially in the entrance examinations. This was a unique case, where a pupil had already won his doctorate and was more qualified and knowledgeable than his teachers. At the age of 25, he was appointed as an interim professor of mathematical physics at the Collège de France, the most prestigious French institution of scholarship, to replace J.B. Biot (1771-1862), the last representative of Laplacian science and of the Société d'Arcueil. After Biot's death in 1862, Bertrand was appointed to the Chair.

Joseph Bertrand became a tutor *répetiteur* in analysis at the Ecole Polytechnique in 1844 (until 1856) and Chair professor in the subject from 1856 to 1895. He thus taught 50 intakes of Polytechnique students, among them some of the most renowned French scientists of the second half of the 19th century, as well as eventually famous representatives of industry, senior administration and the army. Bertrand was an exceptional teacher, brilliant, witty, always clear and precise, avoiding obscurities and limiting his lectures to what he himself could understand perfectly, or at least give the impression of understanding perfectly.

On Sturm's death in 1856, he was elected to membership of the Academy of Sciences. He became its permanent secretary for the mathematical sciences from 1874 until his death. In this capacity, he produced two volumes of academic eulogies, elegant and witty in the style of the period, which secured

his election in 1884 to the Académie Française.

Bertrand's mathematical work was important and occasionally brilliant, but perhaps not profound. It was concerned with branches of mathematics in vogue in the second half of the 19th century, analysis and geometry of course, but also arithmetic and algebra for which he had a strong flair (Bertrand's curves, Bertrand series, Bertrand's postulate). His acknowledged master in all things was Gauss (q.v.), and his models Abel and Jacobi.

From an early age, Bertrand revealed his notable gift as a polemicist. His ferocious and deadly irony rapidly assured him of leadership in the numerous institutions of which he was a member. His natural authority, and an understanding of the subtleties of arbitration, assisted by carefully selected and maintained family alliances assured him of automatic majorities at the Sorbonne, the Academy of Sciences, the Ecole Polytechnique and the Collège de France. Bertrand was the brother in law of the benevolent Charles Hermite (1822-1920), who described him as "ill-disposed and ill-natured; with him one could not be totally sure of anything". Bertrand was also an uncle by marriage of Paul Appell (1855-1930)- who was himself the father in law of Emile Borel (q.v)- as well as of Emile Picard (1856-1941). For the best part of half a century, Bertrand almost singly acted as caretaker of the French mathematical sciences; there is no other example of such power, except for that, even greater from all points of view, of the earlier Siméon-Denis Poisson (q.v.).

Bertrand early became interested in the calculus of probabilities, which he taught for 40 years at the Ecole Polytechnique and for several years at the Collège de France. It was he who translated Gauss' papers on the Method of Least Squares into French; he loved Gauss' algebraic clarity, in contrast to Laplace's (q.v.) analytic obscurities. He was interested in combinatorial analysis, whose profound aesthetics he was one of the first to appreciate in France: his students Emile Barbier and Désiré André later contributed to its development. It was without any doubt Bertrand who popularized the Method of Expectations in which the expectation of the indicator function of an event, rather than its probability, is calculated. His large treatise *Calcul des probabilités*, the conclusion of 40 years of carefully revised teaching, went through two editions (1888 and 1907). Its success is largely due to the enormous gap in France between the probability books of Poisson of 1837 and of Cournot(q.v) of 1843, and his own, filled in part only by the book of H. Laurent in 1873. Bertrand's book is of great interest on more than one count, and not only because it served as a reference text for all

mathematicians interested in the theory of probability at the beginning of the 20th century. Apart from some regrettably narrow-minded views, as for example on Condorcet (q.v.), Canard and Cournot, that is: against the use of mathematics in economics and the human sciences, this book is remarkably well written, and could well serve to exemplify a certain type of French intellect at the end of the last century. In it, one can certainly find the famous Bertrand paradox on the different ways of drawing a chord at random in a circle; this is a paradox which Bertrand in his handwritten lecture notes for the Ecole Polytechnique constructed in stages as a transformation of the famous Buffon needle problem. More importantly, his Chapter VI on the gamblers' ruin is at the very foundation of modern research on the theory of Brownian motion and the sums of independent random variables. Bachelier (q.v.) found much inspiration in it, and through him modern probabilists. Poincaré, Hadamard, Borel and Paul Lévy studied Bertrand's treatise, for which they always showed the greatest respect.

On the other hand, Bertrand remained resolutely hostile to Laplacian or Bayesian statistical theory: the chapters on these topics in his book are totally sceptical. The "probability of causes" was, to his mind, too arbitrarily tied to the prior distribution to be useable in practice under any circumstances. However, his comments are often acute and lucid. Bertrand's hostility to Laplace extended to Bienaymé (q.v.), who near the end of his life expressed small regard for some of Bertrand's contributions. Bertrand's book helped bury the memory of Bienaymé's contributions by an inadequate and negative treatment. Bienaymé's probabilistic alter-ego, Chebyshev (q.v.) is not mentioned once. In a letter to Chebyshev, Catalan writes with uncharacteristic mildness: "Quel drôle de livre! [What a funny book!]". There were several clashes between Catalan and Bertrand, but few were prepared to take Bertrand on. However, Bertrand did not always emerge unscathed; the best known instance is "L'Affaire Carton" of 1869 when he presented for publication in the *Comptes Rendus* of the Academy a paper by one Jules Carton which purported to give a proof of Euclid's parallels postulate. Liouville and Bienaymé, and then Darboux, Beltrami and Hoűel, opposed publication for obvious reasons. Darboux, a former pupil of Bertrand, finally succeeded in persuading him his position was wrong.

The only areas of application of the calculus of probabilities in which Bertrand was apparently interested were artillery accuracy (in which he was the forerunner of Gaussian statistics, correlation, axes of inertia), topics in geodesy (although his chapters on the Method of Least Squares were not to-

tally convincing), and particularly problems of insurance, certain aspects of which he attempted to clarify. In this respect, Bertrand is very representative of his period and his "School". In fact, during the second half of the 19th century after a period of decline in mid-century, graduates from the Ecole Polytechnique excelled, in the tradition of Poisson, in the probabilistic study of the dispersion of different artillery firings, as can be seen from the lectures of Henry (1894). This even became a specialty of the applied military schools (Metz and later Fontainebleau), with artillery men creating, within the milieu of the military, several modern statistical tools such as chi-square. It was graduates of the Ecole Polytechnique who also transformed actuarial methods. Among them were H. Laurent, and E. Dormoy who developed a coefficient for the study of stability and non-normality in statistical series at the same time as Lexis(q.v.) in Germany. Most of these mathematicians had followed the lectures of Bertrand, who had incorporated the advances of his students into both his teaching and his treatise.

At a time when French science was no longer central in the world, Joseph Bertrand played an arguably indispensable (if somewhat questionable) role, particularly in the renaissance at the beginning of the 20th century of the French school of probability. Fortune,too, played no little part in his ascendancy. According to the concluding lines of Zerner(1991), a concurrence of circumstances ensured that an oligarch became de facto a monarch, even though posterity retains almost nothing of his mathematical work. One of these was that Bertrand arrived at adulthood in a period poor in first rate mathematicians.

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