

## Harold Edwin HURST

b. 1 January 1880 - d. 7 December 1978

**Summary.** Harold E. Hurst, English hydrologist and “Father of the Nile,” is principally known for a remarkable power-law relation that was later interpreted as symptom of “long-run”, or “global” statistical dependence in the Earth sciences.

Hurst, hailed as perhaps the foremost Nilologist of all time and spoken of as “Abu Nil,” the Father of the Nile, spent the bulk of his career in Cairo as a civil servant of the British Crown, then of Egypt. (*Who’s Who*, 1979, p. 1257, and *Who’s Who of British Scientists 1969/70*, pp. 417-418.)

His early training is worth recounting, mostly from his own words and those of his wife Mrs. Marguerite Brunel Hurst. The son of a village builder of limited means, whose family had lived near Leicester for almost three centuries, he left school at age 15. He had been trained mostly in chemistry, and also in carpentry by his father. He then started as a pupil teacher at a school in Leicester, attending evening classes to continue his own education.

At age 20, he won a scholarship that enabled him to go to Oxford as a noncollegiate student. After a year, he became an undergraduate at the recently reestablished Hertford College, and soon switched to a major in physics and worked at Clarendon Laboratory.

His lack of preparation in mathematics was a handicap, but thanks to the interest that Professor Glazebrook took in an unusual candidate who was very strong in practical work, he won a first-class honors degree, to everyone’s surprise, and was asked to stay for three years as a lecturer and demonstrator.

In 1906, Hurst went to Egypt for a short stay that was to last 62 years, of which the most fruitful were after he turned 65. His first duties included transmitting standard time from the Observatory to the Citadel of Cairo, where a gun was to be fired at midday. However, he became increasingly fascinated with the Nile, and his study and exploration of the Nile basin made him well known internationally. He traveled extensively by river and on land, first on foot with porters, then using a bicycle, later by car, and later still by plane. The low Aswan Dam had been built in 1903, but he realized how important it was to Egypt that provision should be made not only for the dry years but for a series of dry years. Irrigation storage schemes should be adequate for every situation, very much, as in the Old Testament,

Joseph stored grain for the lean years. He was one of the first to realize the need for the “Sudd el Aali,” that is, the High Dam and Reservoir at Aswan.

Hurst’s name is linked to a statistical method he initiated, without at all realizing the scope of what he was doing. He used this technique to discover a major empirical law concerning the form statistical dependence takes in geophysics. At first, it seems surprising that anything of the kind could come from an author so poorly prepared in mathematics and working so far from any major center of learning, but at second thought these circumstances may have been vital to both the birth of his idea and its survival. He investigated the Nile using a peculiar method of analysis that follows very literally a recipe found in ancient books on optimum dam design and consists in a ratio  $R(d)/S(d)$ . The numerator  $R$  was defined, in current terminology as the range over a time span  $d$  relative to the cumulative sum of river discharges. The denominator  $S$  was defined as a standard deviation of yearly river discharges. Critics termed  $R/S$  narrow, undocumented, and ad hoc, but in fact this ratio - quite unexpectedly - turned out to be eminently intrinsic to the problem at hand and to have unparalleled qualities. With the computers of today, the calculation of  $R/S$  is easy, but in the mid-1960s, it was viewed as very time-consuming. Therefore, one can imagine the amount of hard work implied in such research before the advent of computers. But of course the Nile is sufficiently important to Egypt to justify comparatively large expenditures (and to preclude forcing Hurst to retire). Not being pressed by time and having exceptionally abundant data at his disposal, Hurst was in a position to evaluate their effect upon the design of the then-future High Dam. He showed that the dams the data demanded must be far larger than suggested by the standard model of stochastic variability of river discharges. Indeed, all earlier writings on this topic led to the expectation that in the long run  $R(d)$  should increase like the square root of  $d$ , whereas Hurst’s extraordinarily well-documented empirical law shows an increase like the power  $H$  of  $d$ , where  $H$  is about 0.7. Hurst adamantly maintained that this finding was significant, despite the fact that no test existed by which its statistical significance could be assessed objectively. Finally, at the ages of 71 and 75, he read two long papers that attracted attention. He was awarded the Telford Gold Medal by the Institution of Civil Engineers in 1957.

In the 1950s and 1960s the statistical profession was not at all prepared to tackle “power-law” relations like Hurst’s. Scaling was an esoteric notion. Hydrologists were advised to manage with Markov or ARMA processes. Unfortunately, the memory those processes required increased with the length

of the sample.

The significance of Hurst's work was revealed through contributions of B.B. Mandelbrot starting in 1965. He first interpreted the power-law as a symptom of underlying scaling, more precisely, of self-affinity with an anomalous exponent  $H$ . The standard Brownian case  $H = 0.5$  corresponds to independent increments, hence no statistical dependence whatsoever. But when  $H$  is not 0.5, the dependence is very strong, positive or negative. It can be called, "long-range," or perhaps better, "global." Furthermore, to show that  $H$  differing from 0.5 is not inconceivable, Mandelbrot proposed that, instead of the Wiener Brownian motion, the cumulative river discharge might be represented by the fractional Brownian motion model, which he devised for this purpose.

The division by  $S$  began as a traditional normalization, not deserving a careful look. But as Mandelbrot showed, it turned out, serendipitously, to be extraordinarily important. The undivided  $R$  is not robust at all, being confined to measure global dependence when the margins are Gaussian or follow other very short-tailed random variables. After division by  $S$ , to the contrary, the ratio  $R/S$  is extraordinarily robust. It gave rise to a new technique, " $R/S$  analysis," which can reveal and estimate global dependence independently of the length of the marginal distribution's tails and remain useful even when the data has an infinite variance.

This is an instance of when a result is truly unexpected, is hard to comprehend, even by those best disposed to listen. New tools are not accepted without struggle.

## References

- [1] Hurst, H.E. 1951. Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, **116**, 770-808.
- [2] Hurst, H.E. 1955. Methods of using long-term storage in reservoirs. *Proceedings of the Institution of Civil Engineers*, part 1, 519-577.
- [3] Hurst, H.E. Black, R.P. AND Simiaka, Y.M. 1965. *Long-term Storage, an Experimental Study*. Constable, London.

- [4] Mandelbrot, B.B. 2000. *Gaussian Self-Affinity and Fractals: Globality, the Earth,  $1/f$ , and  $R/S$* . (Selected Papers Vol. H). Springer, New York. Reproduces and makes easily available the author's and his associates' papers from the 1960s and 1970s.

Benoit B. Mandelbrot