D'ALEMBERT
b. 17 November 1717 - d. 29 October 1783

Summary. D'Alembert's doubts concerning the calculus of probabilities are more pertinent than has long been believed, and were the starting point for the work of Condorcet and Laplace.

D'Alembert was the illegitimate son of the officer Destouches-Canon, and Madame de Tencin. He was abandoned on the steps of the church of Saint-Jean-le-Rond in Paris, and put in the care of a glazier's wife. Initially called Jean Le Rond, he later gave himself the name of J.B. Daremberg and then Dalembert or D'Alembert. In fact his father did not totally abandon him, since he provided for his education. The young man was a pupil at the Collège des Quatre-Nations, where he studied law and medicine, but he rapidly began to concentrate on mathematics. He became a member of the French Royal Academy of Sciences in 1741, as "adjunct astronomer" and soon became known through his *Traité de dynamique* (1743) which based mechanics on a new principle. In the few years until the beginning of the 1750's he published some papers of remarkable originality on fluids, vibrating strings, the precession of equinoxes, and lunar theory. We owe to the young D'Alembert the first equations with partial derivatives, the fundamental theorem of algebra, an essential contribution to the equations for perfect fluids, and the statement of a paradox in hydrodynamics. The style of the young author exhibited a number of characteristics which remained with him for life: the desire to construct theories starting only from a minimum of principles, a demanding nature which admitted nothing without an investigation of principles, a critical study of the link between mathematics and the conditions for its application, and a method of mathematical exposition which was often difficult to follow.

D'Alembert was also given to polemics, inflexible in his priorities and relentless against his adversaries; among these were Daniel Bernoulli (q.v.), Clairaut and Euler, who gave as good as they got.

In 1751 the first volume of the *l'Encyclopédie* appeared, with the preface "Discours préliminaire" written by D'Alembert. He had been made responsible for the mathematical section of this enterprise, and it was in this capacity that he wrote his first contributions to the calculus of probabilities. Following some indirectly critical articles on this topic a scandal broke out about the article on heads or tails, (croix ou pile) in volume IV of the *Encyclopédie*.

D'Alembert contested principles accepted by everyone: he cast doubt, for example, on the fact that the probability of obtaining heads in two trials was 3/4, and suggested that it might be 2/3, arguing that there were in fact only three possible events (the coin is not thrown again if heads occurs at the first trial), namely heads, tails followed by heads, and tails followed by tails, the first two being favourable, while the third was not. D'Alembert was thus provocatively initiating a prickly debate on the question of equally likely events; this debate became even more prickly when the calculus of probabilities was applied to events in real life rather than simple games of chance. In his contribution to the famous St. Petersburg problem, he disputed not only Daniel Bernoulli's solution, in which the fate of a player is evaluated by a criterion other than the mathematical expectation, but also other solutions involving limiting the number of trials in the game, or neglecting smaller probabilities, etc. In other words, D'Alembert doubted not so much the abstract rules of probability, but rather the relevance of these rules to moral, that is human and social, behaviour. He would not tolerate the separation of the calculus of probabilities from its use in concrete examples. He refused to assume without severe appraisal all the simplifying hypotheses which one usually makes without much thought: the assumption that events are equally likely, their independence, the additivity of probabilities, and the linearity of expectation. He also found it difficult to bear not being able to take the effect of time into account in his calculations. In the calculus of probabilities, as also in mechanics and in the rest of science, he contested the principle of proportionality of causes to effects.

D'Alembert developed his doubts and objections in various works until his death, for example in his *Opuscules mathématiques*, and in books accessible to a larger public like his *Mélanges de philosophie*. An ideal case for the concrete discussion of the principles of the calculus of probabilities was provided by the controversy on inoculation which raged in the 1750's and 1760's. A supporter of inoculation, he did not accept the fact that one should measure its advantage simply by evaluating the increase in life expectancy. He was not happy with crude summary measures; he queried those indicators which we call the mean and the median, indirectly requiring a study of the variability. He wanted made explicit the manner of dealing, within the same calculation, with events separated by time; and insisted that one should distinguish the viewpoint of the individual from that of the State. In all these disputes Daniel Bernoulli was D'Alembert's main adversary. Bernoulli was a very skilled analyst, inclined to choose effective simplifications; he constructed

striking and seductive theories, but refused to be dragged into discussions on the principles involved.

D'Alembert is not known for any theorems in the calculus of probabilities, nor what in the twentieth century is referred to as mathematical statistics. The essence of his contributions consists of criticisms, but it would be wrong to believe that he played a purely destructive role. On the one hand, his articles and papers contain propositions, such as, for example, a mathematical theory of inoculation (11th paper in the *Opuscules*), and a comparative criterion for distribution functions (23rd paper). On the other hand, D'Alembert's doubts stimulated the researches of young French mathematicians in the second half of the 18th century, in particular Condorcet (q.v) and Laplace (q.v.). This is clear from their writings: each in his own way, took some difficult questions posed by D'Alembert as a starting point to construct their own theories. Laplace's famous "Mémoire sur les probabilités des causes par les événements", published in 1774, which may in some sense be thought of as the starting point of modern mathematical statistics, is the most obvious example.

D'Alembert's ideas were partially eclipsed in the general area of mathematics by Euler and Lagrange," geometers" who were perhaps more able, but also gifted with a far more readable style, which appeared less archaic to the mathematicians of the 19th and 20th centuries. In the area of the calculus of probabilities, D'Alembert was considered very eccentric. Lacroix, who was among his more polite critics wrote in his treatise on probabilities (1816) "it can happen to the most eminent that they lose their way even in a very simple subject". However, this long-unquestioned judgement has been revised in the decade 1970-80; one may note today that D'Alembert had pinpointed some of the very real difficulties in the calculus of probabilities, particularly in its relation to reality. Looking back, one can see that he did not at the time have the methods to overcome a number of these problems.

D'Alembert took part in many other activities. A close acquaintance of Voltaire and Turgot, like Pascal (q.v.) before him an enemy of the Jesuits, he was one of the better known personalities in the *philosophes* movement. D'Alembert was first a member, and later permanent secretary (1772) of the Académie française, and a friend of Frederick II of Prussia. In this role he was a prime mover in the foundation of the Academy of Berlin and until his death in 1783, played a very important role in the organization of intellectual life in Europe.

His *Oeuvres Complètes* are currently in the course of publication. His

name is known even to students of elementary mathematics through "D'Alembert's test" for convergence.

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