Émile BOREL
b. 7 January 1871 - d. 3 February 1956

Summary. After his celebrated contributions to pure mathematics, Borel militated in support of the calculus of probabilities all his life. He gave impetus to work on almost sure convergence.

Émile Borel was born in Saint-Affrique, in the department of the Aveyron (southern Massif Central), France, and died in Paris. He began his studies at the private school run by his father, the pastor of Saint-Affrique, and continued them at Montauban and then Paris, where he made a great impression with his scholastic successes.

At the age of 18, he won the first prize for mathematics in the 'Concours general' (competitive examination) and was selected as the top candidate at both the École Polytechnique and the École Normale Supérieure, like Darboux, Picard and Hadamard before him. On the advice of Darboux, he chose to attend the second. This was considered an ideal career path in the closed world of French Universities at the end of the 19th century. He attended Hermite's and Poincaré's courses at the Sorbonne; both were to have a profound influence on his scientific life. It was due to Hermite that he was initiated "into the redoubtable and sacred mysteries of the divinity of number"; this grew into a deep love of mathematics, a youthful love which would never leave him. From Poincaré he absorbed an openness of mind, a feeling for the unity and universality of scientific culture at the highest level, the "objective value of Science", even though Borel soon found himself opposed to Poincaré's fundamental scepticism. Poincaré, knowing everything about everything, knew better then most "that we know nothing".

Like many young mathematicians of his generation, Borel was deeply influenced by both Cantorian "romanticism" and Weierstrassian "discipline". In 1894 he defended a dazzling thesis in which he used the Cantorian theory of (linear) sets which are now called after him, to break the confines into which Weierstrass had enclosed analytic function theory, and showed the potential richness of the concept of a set of points of measure zero in the theory of functions. Borelian theory of functions, which he taught the young 'Normaliens' at the end of the 19th century, including Baire and Lebesgue, was soon to infiltrate all real analysis.

In 1893, even before defending his thesis, he was appointed maître de conférences (lecturer) in the Faculty of Sciences at Lille, and in 1897 at the École Normale Supérieure, and then the University of Paris in 1904. In 1909 he became a full professor appointed to a Chair in the Theory of Functions specially created for him. At the same time, he held the post of Assistant Director of the École Normale Supérieure between 1910 and 1920. He remained a professor in the Faculty of Sciences at the University of Paris until his retirement in 1941.

In April 1906, in *La Revue du Mois*, the monthly periodical which he had just founded, Borel defined in a specific manner the new scientific programme which he intended to follow thereafter, namely "The practical worth of the calculus of probabilities". Borel at once disposed of one of the principal interpretations of the calculus of probabilities, which the 19th century had accepted as a last resort . This was the double meaning (double nature or double origin) of probability, in that it could be "subjective" or "objective". Borel wrote "There is no difference in nature between objective and subjective probability, only a difference of degree. A result in the calculus of probabilities deserves to be called objective, when the probability is sufficiently large to be practically equivalent to certainty. It matters little whether one is predicting future events or reviewing past events; one may equally aver that a probabilistic law will be, or has been, confirmed."

Borel's eventual position on the double nature of probability evolved very slightly after the appearance of Keynes' (q.v.) (1921) treatise on probabilities, in which Keynes adapted a subjectivist-logical position. Borel distanced himself from it, judging that "if Keynes is correct from the theoretical point of view, Poincaré's opinion is practically justified and exact." Borel's thinking was no doubt fairly close to this austere dualism: theoretical subjectivity-practical objectivity.

Simultaneously Borel published, one after the other, two mathematical papers on the calculus of probabilities. The first, in 1905, explained how Borel measure allowed one to extend and make more precise the calculation of "geometric probabilities". The second was more ambitious: it attempted to show that the calculus of probabilities, while certainly playing an interesting role in human affairs, could also explain the paradoxes of statistical physics. Thus, even adopting Poincaré's viewpoint of science pursued for its own sake, the calculus of probabilities could shed light on some of its darkest shadows, namely those surrounding the kinetic theory of gases.

The only possibility open to a scholar was to reason probabilistically and to seek, as in a game of heads or tails, those motions among all the possible ones which were the most probable, particularly as these rapidly become the only ones physically possible. As is well known, Borel put forward the parable of the monkey typists to give an idea of the extreme improbability of a small deviation from the most probable configurations. It is easier for an army of monkeys typing on the keyboards of typewriters, to compose all the books in all the libraries of the world, than for a mixture of two initially separate gases to return, by the random collision of their molecules, to their initial states.

The *Revue du Mois* having proclaimed (the first in the world to do so) both the practical and scientific values of the calculus of probabilities, there remained for its director only to pursue the good fight wherever he could. Borel did so initially at the Sorbonne where he gave his first course on the calculus of probabilities in 1908–1909, the same year that he had been appointed as full professor to the Chair of the Theory of Functions. In the same year 1909, Borel took an active part in the Congress of the International Statistical Institute (ISI) held in Paris, campaigning to regain a place for the application of the calculus of probabilities to statistics. Until then, the ISI had concentrated on official statistics.

In the same year, 1909, Borel published a classical work in the mathematical theory of probability, on “denumerable probabilities”. The so-called Borel-Cantelli Lemma originates in this work. In this paper, for the first time, it was stated and (almost) proved that the sequence of relative frequencies of tails, in an indefinite sequence (“denumerable”) of games of heads and tails, converges to the probability of a tail in a single trial, except for a set of games of probability zero. This statement was to accelerate appreciably the rather slow development of probability theory, and may without exaggeration be considered the starting point of modern probability, giving it a new dimension, embodied in almost sure convergence, and Borel sets of probability measure zero.

Borel’s article was also read by mathematicians, partly because it contains statements on the theory of numbers which were of immediate current mathematical interest, particularly the first among them. This was that the set of real numbers on the unit interval consists almost surely of normal numbers, the frequencies of each of the digits in their expression (in any base whatever) being asymptotically equal with probability 1, if the digits are drawn at random one after the other. As Borel said, and Lebesgue further explicated to him, the set of normal numbers has the Borel measure 1. This article contains a second application to the theory of numbers, an account of the properties of the sequence of digits in the continued fraction expansion of a number drawn at random from the unit segment.

Did Borel train a French "school" of probabilists? Any reply to this question must be qualified. As a humanistic scholar, Borel elevated the calculus of probabilities to the highest level, possibly too high a level; as a mathematician, he set it too low. Borel thought that mathematicians should first train themselves in mathematics by studying the great mathematical theories (for example, the theory of functions). He thought that the calculus of probabilities was somewhat remote from the "great problems", such as those of Hermite, Darboux or Poincaré. What is more, and rather paradoxically, Borel was not very interested in the axiomatisation of probabilities.

The First World War oriented Borel's activities even more towards applications. France was in danger, and the scientific community mobilised itself to defend and protect it from "German barbarism". Painlevé, who had entered the political arena after the Dreyfus affair, was elected deputy for Paris in 1910, and was entrusted in 1915 with the direction of the large ministry of Public Education, Arts and Inventions relevant to National Defence. Within the latter, he created the Directorate for inventions in the service of National Defence, and named Borel "Head of the Technical Office" . When Painlevé was Minister for War from March to September 1917, Borel was Director of the Ministry's Technical Services, and when Painlevé rose to the Presidency of the Council at the most tragic moment of the war in the autumn of 1917, Borel was the general Secretary of the government, entrusted with all missions, both possible and impossible, including those connected with the scientific aspects of the war. He later became deputy for the department of the Aveyron from 1924 to 1936, mayor of Saint-Affrique from 1929 to 1941 and from 1945 to 1947, and Minister for the Navy in 1925.

Borel's political activities did not prevent him from devoting some of his time to new "flashes" of inspiration in the theory of probability, particularly on the theory of games (1920) or queues (1942). It was also he, together with Lucien March, Director of the Statistique Générale de la France (French Statistical Bureau) and Fernand Faure, holder of the Chair of Statistics in the Faculty of Law of the University of Paris (the first such chair in France, founded in 1892), who was responsible for the creation of the Institut de Statistique de l'Université de Paris, an institute serving the four faculties of the University of Paris (Law, Literature, Medicine, Sciences), which starting in 1922 was to train the competent statisticians necessary for Borel's France.

At the same time, Borel launched a new series with Gauthier-Villars (the publishers) of which he was the sole editor. This was the *Traité du Calcul des*

Probabilités et de ses Applications, which consisted of 5 volumes and 18 fascicles published between 1925 and 1939, five of these being written by Borel himself. This was an important work, which fell fairly rapidly into relative obscurity after the war. The great treatises on probability published directly after the war, namely those of Cramér (q.v.), Doob, Feller, Loève, ..., resolutely adopted an expository style which set the pattern for the mathematical works of the time and thereafter: explicit axioms, precise enunciations, full proofs in a logical order. Yet one knows very well since Cournot (q.v.), that this is neither the rational nor the practical order, which was why Borel had rejected it.

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