**Anderson Gray McKENDRICK**
b. 8 September 1876 - d. 30 May 1943

**Summary.** Anderson McKendrick, a British medical doctor and capable mathematician, pioneered many discoveries in stochastic processes. He also collaborated with W.O. Kermack in path-breaking work on the deterministic model for the general epidemic.

The use of mathematical methods in epidemiology had become well established by the early 20th century. Among the pioneers were British scientists, such as Brownlee, Greenwood, Hamer, Ross and Soper. But none appears to have been as influential in this field of research as Anderson Gray McKendrick.

McKendrick was born in Edinburgh in 1876, the fifth and last child of John Gray McKendrick FRS and Mary Souttar. A few months after his birth, his father was appointed to the Chair of the Institutes of Medicine at the University of Glasgow, later renamed the Chair of Physiology in 1893. After graduating in medicine from the University of Glasgow in 1900, McKendrick joined the Indian Medical Service (IMS), achieving the top marks in their entrance examination. Before taking up his medical post in India, he was sent on special duty with Sir Ronald Ross in Sierra Leone to study anti-malarial operations, and the two returned to Britain on the same ship. Ross was keen on the mathematical modelling of malarial transmission, and exerted an important (and generously acknowledged) influence on McKendrick's later work in mathematical epidemiology. For a more detailed account of these and other events in McKendrick's life, the reader is referred to Aitchison and Watson (1988).

It was customary for new recruits to the IMS to undertake a period of military service, and Lieutenant McKendrick spent 18 months in Somaliland during the expedition against the Mahdi of the Sudan. Among his activities, he succeeded in building a tank to contain safe water for his troops. On his return to India, he was posted to Nadia in the Bengal, where he further demonstrated his interest in public health by attempting to reduce the occurrence of dysentery in the district jail.

In 1905, McKendrick was appointed to the Research Department of the Government of India, and posted to the Pasteur Institute at Kausali in the Punjab. He became interested in rabies and, in 1907, wrote a memoir on anti-rabic immunisation with W.F. Harvey. His routine at Kausali included a systematic study of Mellor's *Higher Mathematics for Students of Chemistry and Physics*.

McKendrick served in India for several years, rising to be a Lieutenant-Colonel in the IMS; he worked in the Pasteur Institute at Coonoor in Southern India, briefly became Statistical Officer with the Government of India at Simla, and returned to Kausali as Director of its Pasteur Institute. His time there coincided with the First World War, and the pursuit of research, which was his principal interest, proved difficult; he worked on problems of epidemiology, including the mode of carriage of infections. Having contracted sprue, a tropical disease characterized by diarrhea, emaciation and anemia, he was retired from the IMS to Britain in 1920 to recover, and settled in Edinburgh with his wife and four children.

Shortly after, he was appointed to the post of Superintendent of the Laboratory of the Royal College of Physicians of Edinburgh, an institution partly supported by the Carnegie Trust. The young W.O. Kermack, later to become Professor at Aberdeen and an FRS, was appointed at the same time to a post in biochemistry in the Laboratory. Though blinded in an experiment in 1924, this did not prevent him from continuing his work.

Anderson McKendrick held his position at the Royal College of Physicians for over 20 years. He organized the Laboratory for large scale routine medical examinations, while also creating a new department of medical chemistry, and a specialised department of medical research. He held the view that the Laboratory should be research oriented, with the senior staff freed from teaching and administration. He pursued his own interests there, collaborating with W.O. Kermack in many of his epidemiological papers, in particular on the deterministic model for the general epidemic.

McKendrick's 58 publications discuss a variety of topics: medical (on rabies and malaria), statistical (on classification and age distribution), demographic (on death rates in Britain and Sweden), and increasingly after his appointment at Edinburgh, on anti-rabies treatment and different aspects of mathematical epidemiology. He recognized that his mathematical papers would not be easily understood by medical colleagues, and attempted in general lectures to popularize his subject for their benefit. He was a sensitive and kindly leader of men, who was congratulated by Professor J.T. Wilson of Cambridge (see W.F.H., 1943) for the Carnegie Trust on the "spirit of comradeship pervading the entire laboratory staff", as well as the laboratory's "achievements in a number of important fields of research". McKendrick retired in 1941 and died in 1943 at Carrbridge in Inverness-Shire.

In the development of mathematical epidemiology, Hamer in 1906 had been the first to suggest a concept of mass action (homogeneous mixing) for London measles epidemics. Ross in 1911 independently made use of this principle in the appendix of his book on malaria, and McKendrick developed it further, both individually (1912), and in his joint work with Kermack (1927). McKendrick, inspired as he was by Ross's work on epidemics, is remarkable for three major contributions to epidemic modelling. The first is contained in a paper published in the Proceedings of the London Mathematical Society (1914) in which he effectively described the Poisson Process, derived equations for the pure birth process, and a particular birth-death process. He also formulated the differential equations for the deterministic general epidemic, which he later elaborated with Kermack (1927). Already in this early work, he had anticipated many of the later discoveries in the area of stochastic processes.

The second is contained in a paper in the Proceedings of the Edinburgh Mathematical Society (1926). In this he considered two types of one-dimensional compartmental models, and examined a semi-stochastic case; he was able to conclude from this that a Poisson distribution in observed cases of illness meant that there was no infectious process involved. In his Example 2 on a cholera epidemic in India, data was collected on the numbers of households in which $0, 1, 2, 3$, and 4 cases of infection had been observed. The Poisson distribution was a good fit except for the 0 case, where the expected Poisson value was very much smaller than the observed value. This suggested to McKendrick that the primary cause of infection was probably a particular well from which the majority of infected households drank; on further investigation, this proved to be the case. He also went on to study the two-dimensional compartmental case. One may regard some of his results for the numbers $v$ of individuals in compartment $(x, y)$ as foreshadowing the differential $xy$ equations for the probabilities of the general stochastic epidemic. For discussions of the results in these papers, the reader is referred to Irwin (1963), Gani (1982) and most recently Dietz's (1997) introduction to the 1926 paper.

His third contribution with Kermack (1927) is perhaps the best known of all: in it, they set out the equations for the deterministic general epidemic in a closed population of size $N$. This is subdivided into $x(t)$ suceptibles, $y(t)$ infectives and $z(t)$ immunes, where $x(t) + y(t) + z(t) = N$. Infection spreads through homogeneous mixing, so that

$$dx/dt = -bxy, dy/dt = bxy - gy, dz/dt = gy,$$

where $b$ is the rate of infection, and $g$ the rate of removal of infectives from the population. These equations may be found in the earlier work of Ross in 1915 and Ross and Hudson in 1917 as can also the consequent threshold result, so that strict priority would attribute the results to these authors. Kermack and McKendrick (1927) themselves acknowledge this earlier work but claim to go a stage further. Epidemiologists have commonly regarded the solution of the differential equations and the formalisation of the threshold theorem as Kermack and McKendrick's.

An explicit solution to the equations was obtained by them using a quadratic approximation to the exponential; this then leads to the so-called Kermack-McKendrick threshold theorem. Briefly, its results state that the epidemic will spread only if $x(0) > g/b$, and that under certain simple conditions, there will always be some susceptibles left at the end of the epidemic. Very roughly, if $x(\infty)$ susceptibles survive, then

$$x(\infty) = x(0) - 2g/b,$$

where this must be positive.

McKendrick's work of 1926 and Kermack and McKendrick's deterministic results of 1927 were extended in full detail to the stochastic general epidemic by Bartlett (1949), and a threshold theorem for the stochastic case derived by Whittle (1955). A large part of the subsequent development of epidemic modelling stems from their seminal paper, which together with Ross's pioneering work can be regarded as the foundations of modern mathematical epidemiology.

*Acknowledgement.* My thanks are due to Professors K. Dietz and H. Heesterbeek for their help in preparing this article.

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