Jarl Waldemar LINDEBERG
b. 4 August 1876 - d. 24 December 1932

Summary. Finnish mathematician and statistician, Lindeberg is best known for his important proof of the central limit theorem.

Jarl Waldemar Lindeberg was son of a teacher at the Helsinki Polytechnical Institute; the family was well-to-do. He was aware early of his mathematical talent and interest. Having taken his candidate's degree in 1897, he went for a year to study in Paris, with the support of an aunt. Provided from there with a theme - apparently essentially his own choice - for his doctoral dissertation, on partial differential equations, he went for the following winter with his family to Corfu, sharing his time between scientific and social occupations. In December 1900 he was ready to defend his thesis, with Ernst Linde1öf as the appreciative opponent. In spring 1902 he was appointed docent and three years later adjunct of Mathematics. In 1919 he was awarded the title of professor. Being economically independent, and feeling the position of adjunct best to his taste, he never cared for an ordinary chair. Apart from his teaching at the university he also taught at the Technical University 1911-18, and in his younger days in a Finnish secondary school. Among his pupils he found his future wife, Inez Beeker, whom he married in 1905, barely a year after her graduation.

Lindeberg remained interested in the problems of school teaching, and represented Mathematics in the Matriculation Examination Board from 1902 to 1918. The problems that he gave were expressly aimed at testing not only knowledge but also the ability to think; afterwards, he often had to defend them against enraged school teachers. What was wrong, according to him, was not the examination in itself, but rather the widespread tendency to require this examination for professions that had nothing to do with true academic studies.

He was elected member of the Society of Sciences in 1909 and of the Finnish Academy of Sciences in 1919. In the latter he acted for many years as treasurer.

Lindeberg's scientific activity comprises several different periods. Having started out with partial differential equations, he shifted to the calculus of variations, which occupied him from about 1904 to 1915. Around 1918, there is a short interlude given to the theory of functions. His later years, until his early death in 1932, were devoted to the fields in which he was to become most renowned, viz., probability and statistics. A first impulse in this direction he probably received when he was appointed to the administrative board of a short-lived life insurance company in 1912; his main achievment was actually its sound liquidation. He was later an active member of the Finnish Actuarial Society. From 1916 he regularly lectured on probability, and was soon considered a national expert in that field; in 1925, his adjunctship was redefined so as to cover, in particular, the calculus of probability and its applications. At about the same time representatives of several scientific associations requested him to write an elementary text-book on the subject, and in 1927 he published his *Todennakoisyyslasku* (Calculus of Probability with Applications to Statistics) which presupposes no calculus. The ultimate objective is to provide research workers in various fields with tools for critical judgement.

Lindeberg's scientific profile is that of an independent and critical research worker, a man who is able to distinguish between essentials and unessentials, and feels an urge to get to the bottom of problems. These qualities were of particular importance in his dealing with probability and statistics - fields on the border line between pure and applied Mathematics, and fields in which the fundamental notions were at that time just about to crystallize. For instance, in choosing descriptive characteristics for distributions, Lindeberg, with typical common sense, first of all wanted them to give easily understandable answers to simple questions.

In his thinking habits, he seems to have been a lone wolf; he worked best when alone at his desk, trying to find out things for himself rather than consulting predecessors. According to Lindelöf, Lindeberg always found it laborious to study other people's writings. He spoke with much modesty and joking irony of his own relationship to science. "You see", he once said, "owning a country place with forest and farming land, in Helsinki I can defend my laziness by saying that I am really a farmer, and in the country by claiming really to be a scientist". In his earlier years, he had serious doubts about his ability.

As an academic teacher, Lindeberg was painstaking and helpful but somewhat dry. In private life he was a friendly and sociable person, with wide interests, and a large circle of relatives and friends among whom he was much loved.

Lindeberg's first and most famous contribution to probability and statistics was his new proof of the *central limit theorem*. That is, roughly speaking, the sum of a large number of independent random variables is approximately normally distributed. A proposition to this effect was first formulated by Laplace in 1812. The first rigorous proof of it was given by Ljapunov in 1901. The conditions imposed in his proof are fairly wide, but it suffers from the drawback that it requires the existence of the third absolute moments of the component distributions. When Lindeberg first attacked the problem he was not aware of Ljapunov's proof, only of a much weaker result by von Mises (q.v.). His first paper on the matter, *Über das Exponentialgesetz in der Wahrscheinlichkeitsrechnung* (Ann. Acad. Sci. Fenn., 1920) essentially leads up to Ljapunov's results, but uses entirely different methods. Whereas Liapunov had used the characteristic function, Lindeberg's proof, although rather intricate, is in principle elementary: using the ordinary convolution formula, he step by step compares the distributions of the cumulative sums of (a) the arbitrary random variables under consideration, to those of (b) normal variables with the same variances as in (a). An ingenious device is to start off both series with an auxiliary normal term. Two years later, having learnt about Ljapunov's proof, Lindeberg modified and improved his own, in *Eine neue Herleitung des Exponentialgesetzes in der Wahrscheinlichkeitsrechnung* (Math. Zeitschr., 1922). His ultimate result, which goes beyond Liapunov's, can in a slightly modified version be formulated as follows: Let $X_1, X_2, ...$ be independent random variables with means 0, with variances ${\sigma_i}^2$, and with cumulative distribution functions $F_i(x), i = 1, 2, ...$ Let ${s_n}^2 = {\sigma_1}^2 + ... + {\sigma_n}^2$ be the variance of the sum $S_n = X_1 + X_2 + ... + X_n$. Then a *sufficient* condition for the c.d.f. of S_n/s_n approaching uniformly the normed c.d.f. with mean 0 and variance 1, is that the "normed tail sum" of the variances tends to zero, i.e. that the following condition (L) holds:

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{s_n}^{-2} \sum_{k=1}^{n} \int_{|x|>ts_n} x^2 dF_i(x) \to 0
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as $n \to \infty$ for each $t > 0$. In 1935, Feller proved that the criterion is, in a sense, the sharpest possible under the plausible restriction to situations in which $s_n \to \infty$, $\sigma_n/s_n \to 0$ the *Lindeberg condition* (L) is actually *necessary.*

Once Lindeberg had become known as an expert on probability, he was approached as consultant in various fields of application: biology, medicine, linguistics, and above all, forestry. He was thus led to an active interest in mathematical statistics, a branch of science which at that time went through a most dynamic evolution, launched by the British statisticians K. Pearson (q.v.) and R. A. Fisher (q.v.). In far-away Finland, the impact of this development was hardly yet to be noted. While one of the main objectives of the British school was the derivation of exact distributions (like the χ^2, t, and F laws) for sample statistics connected with normal distributions, Lindeberg essentially stuck to the classical "large sample" technique, i.e., the approximate calculation of mean errors on which statistical inference could be based if there was enough material to warrant normality. With his characteristic urge for common sense in the applications, Lindeberg was particularly interested in the choice of sensible statistics to describe relevant properties of the sampled population; that is, statistics that gave simple answers to simple questions and were intuitively explainable to laymen.

In a paper read at the Scandinavian Congress in Copenhagen, *Uber die Korrelation* (1925), Lindeberg objects to Pearson's correlation coefficient which "does not answer any natural question", instead, he supports an association statistic called "percentage of correlation", proposed by Esscher and equivalent to what has later been called *Kendall's tau*; its mean error is announced, and in a later publication derived. Several of his papers are actually early contributions, probably not much noted, to the theory of *non-parametric methods* in statistics.

Lindeberg, also became interested in linear multivariate analysis. His paper *Zur Korrelationstheorie* (1929) is concerned with partial regression and correlation coefficients, and notably with the mean errors of the corresponding sample statistics; in a subsequent paper there is a more streamlined treatment of the same subject.

The only consultation work of Lindeberg that led to publications was his dealing with forestry but here his contributions were of lasting importance. They are published in two small papers, *Über die Berechnung des Mittelfehlers des Resultates einer Linientaxierting* (1924) and *Zur Theoria der Linientaxierung* (1926), both in *Acta Forestalia Fennica.* They are concerned with line transect sampling procedures for estimating such quantities as (say) the lumber volume p per hectare, taken in the mean over a large area. In the second of the papers, it is interesting to note an attempt at small sample theory: Lindeberg actually derives Student's distribution, presumably wholly unaware of the achievements of the contemporary British school.

Bibliography

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of the article on Lindeberg in this book.]
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