

## Maurice FRÉCHET

b. 10 September 1878 - d. 4 June 1973

**Summary.** Fréchet, one of the founders of modern analysis, also made various original contributions to the probability calculus. He played an important role in the organization of a European scientific community in the area of probability and statistics.

René Maurice Fréchet was born at Maligny in the département of the Yonne, into a Protestant family, which moved to Paris soon after. As a pupil at the Lycée Buffon, M. Fréchet was taught by Jacques Hadamard (1865-1963), before the latter left Paris for Bordeaux in 1894. In 1900, he was admitted to the École Normale Supérieure, the most selective of Parisian Institutions of higher learning. In 1906, he defended one of the most dazzling French theses in mathematics of his time. In it, Fréchet defined the concept of an abstract metric space, and within this framework demonstrated the validity of Weierstrass's theorem. Fréchet's thesis marks the beginnings of analysis on appropriately structured abstract spaces, one of the principal new directions in analysis of this century (Arboleda, 1980; Taylor, 1982-1987). Throughout his life, Fréchet retained his taste for the most general theories and concepts, and some of those which he introduced were real strokes of genius. Sometimes reproached for not having proved truly "difficult theorems", he is often credited only with introducing new concepts, generalizing and resetting problems in a broader new framework within which he could prove "easy theorems" enunciated for his own interest. It is, however, to Fréchet in particular that we owe the first theory of integration with respect to an "abstract measure", in 1915. Kolmogorov was to make use of this to axiomatize probability theory, citing "his master Fréchet" explicitly as his source. Whatever one's views, Fréchet's scientific activity over a period of 60 years was considerable, amounting to over 300 publications covering all areas of pure and applied mathematics, and including about ten important books.

After a period as a schoolteacher at the Besançon (in 1907) and Nantes (in 1908) Lycées, he was appointed lecturer in the Faculty of Sciences at Rennes. Then in 1910, he became Professor in the Faculty of Sciences at Poitiers, replacing Henri Lebesgue, who had been appointed to Paris. Both appointments were due to the direct intervention of Borel (q.v.). After the First World War, during which he served as an Anglo-French interpreter,

Fréchet was appointed Professor in the Faculty of Sciences at the newly liberated City of Strasbourg, showcase of French science. Actuarial studies and statistics had been taught at Strasbourg for a long time, within the framework of the German university, associated with names such as Lexis (q.v.) and von Bortkiewicz (q.v.). It was here that Fréchet, conscious of his national and international role, and meticulous in the discharge of his duties, began to teach applied mathematics, statistics, actuarial studies, and nomography. His first statistical papers date back to these years. It was entirely to be expected that Fréchet should be called to Paris in 1928, when the Institut Henri Poincaré (IHP) was created, to develop the teaching of probability under the leadership of Borel, its director and the holder of the Chair in the Calculus of Probabilities and Mathematical Physics at the Sorbonne. It should be noted that Borel held the directorship of the IHP from 1928 until his death in 1956. Apart from Borel, Fréchet was at the time the only French university academic of international renown who was interested in recent developments in probability and statistics. Paul Lévy who had just published a remarkable book on the subject in 1925 (Lévy, 1925) was a Professor at the École Polytechnique and never held a university position.

Fréchet was first appointed lecturer in probability at the Sorbonne's Rockefeller Foundation, and then from the end of 1928 as Professor (without a Chair), was promoted to the tenured Chair of General Mathematics in 1933 and to the Chair of Differential and Integral Calculus in 1935. Finally, at the start of 1941, he succeeded Borel in the Chair of the Calculus of Probabilities and Mathematical Physics until his retirement in 1949. In 1928 (and at least until 1935) Fréchet was also put in charge of lectures at the École Normale Supérieure. It was in this capacity, with Borel's blessing, that he directed a sizeable number of young mathematicians towards research in probability, in particular Doeblin, Fortet, Loève, Ville, and others.

As soon as he reached Paris in 1928 Fréchet directed his research towards the new "theory of chain events", namely Markov (q.v.) chains. and published the first mathematical synthesis on this topic in 1938. It was also Fréchet who initiated the study of "random elements" taking values in the most general spaces.

In the statistical field, the usual evaluation of his works was not always enthusiastic, to which following severe comment of Harald Cramér<sup>1</sup> attests:

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<sup>1</sup>"Half a century with probability theory: some personal recollections", *The Annals of Probability*, 4(1976), 509-546 (see p. 528).

“In early years Fréchet had been an outstanding mathematician, doing pathbreaking work in functional analysis. He had taken up probabilistic work at a fairly advanced age, and I am bound to say that his work in this field did not seem very impressive to me.”

Nevertheless, Fréchet was the author of several interesting statistical papers, among them one on the one-dimensional Cramér-Rao inequality, some contributions in econometrics, and in spatial statistics.

His statistical researches came from a critical reflection on the theory of errors. In a series of lectures, papers and discussions, particularly with Paul Lévy, between the two World Wars, Fréchet mainly attacked the excessive hegemony of the Gaussian distribution in all areas of application of the theory of errors, and provided alternative solutions. He demonstrated the usefulness of the Laplace probability density,  $(exp - |x|)$ ,  $-\infty < x < \infty$ , and in connection with it, the use of the median instead of the mean. It was within this framework that he wrote his comments on the *l'homme-médian*, in contrast, presumably, to Quetelet's (q.v.) *l'homme-moyen*. He continued his deviationism in 1926, constructing a theory of errors based on an alternative composition of elementary errors: instead of adding them, he proposed taking their maximum. Using an analogue of P. Lévy's argument in his work of 1925, Fréchet proved some fundamental results for the statistics of extreme values, in particular concerning one of the three asymptotic max-stable distributions. He took up the topic of extremes again in 1947, on the occasion of the jubilee of Richard von Mises (q.v.), another researcher in this area. Meanwhile, he gave his unflagging support to Gumbel's (q.v.) research on extremes, which the latter had begun after his arrival in France in 1932/33.

Other foci of Fréchet's interest in statistics were closely tied to Gumbel, as their rich correspondence indicates: the concentration of incomes, the correlation coefficient, contingency tables with fixed marginals, etc. (Hertz, 1997).

A recently rediscovered insight was what has been named Fréchet Optimality in the theory of probability inequalities (Seneta and Chen, 1996)

Fréchet was a member of the International Statistical Institute from 1931, Honorary Life Member from 1959, and Vice President from 1960. It was he who chaired the first four international conferences devoted entirely to probability and its applications in Geneva (1937), Lyon (1948), Paris (1949), Amsterdam (1954).

For about 30 years, from the mid 1920's to the end of the 1950's, Fréchet acted as an intermediary for communicating information in probability and

statistics, as his varied correspondence with scholars in many countries, preserved in the Archives of the Academy of Sciences, attests. In particular, this repository contains some of the rare surviving letters of Paul Lévy before the Second World War. An open individual, Fréchet worked after the Second World War for the integration of scholars from the Soviet block into the International Statistical Institute, for the peaceful union of peoples, and the spread of Esperanto as a universal language. Some of his works, particularly in Japan, were published in Esperanto.

Fréchet was elected to the Academy of Sciences of the Institut de France in 1956, occupying the seat left vacant by Émile Borel's death.

## References

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