

Pierre de FERMAT

b. 17 August 1601 - d. 12 January 1665

Summary. Arguably the greatest French mathematician of the 17th century, Fermat was instrumental in giving impetus, with Pascal, to the theory of probability.

Fermat, of Basque origin, was born at Beaumont-de-Lomagne, near Montauban in Gascony, the son of Dominique Fermat, second consul of the town, and his wife Françoise de Cazeneuve. His family were leather merchants. He received the elementary part of his schooling, of which little is known, at home and in a local Franciscan school. In 1631 he received his baccalaureate in law from the University of Orléans, having studied, it is thought, in Toulouse and Bordeaux. He worked as a lawyer in the local parliament in Toulouse, becoming Counsellor (*Conseilleur de la Chambre des Requêtes du Parlement*) in 1634. At about this time he married Louise du Long, and the aristocratic *de* was prefixed to his family name. Finding time outside of his legal profession, which necessitated maintaining a certain reserve, he became known through his correspondence for his erudition in the humanities and the classics incorporating a profound knowledge of Greek and Latin, a mastery of modern European languages and research in mathematics. He was created King's Counsellor, still in Toulouse, in 1648.

Despite his intellectual brilliance he was reputedly a quiet, friendly and kindly man, whose successful public life was paralleled by a happy family life. His two daughters became nuns. His son, Samuel, a writer, edited his father's work, published as a two-volume *Opera mathematica* in 1679. Fermat died at the age of 63 (his tombstone says 57), at Castres. There appears to be no definitive biography.

The correspondence between Fermat and Pascal (q.v.), between July and October 1654, which is regarded as laying foundation for the mathematical theory of probability, came about as follows. The unofficial "Academy" which preceded the founding of the Académie des Sciences de Paris in 1665 was that founded by the Abbé Marin Mersenne (1588-1648), indefatigable correspondent on mathematical and scientific matters of his time. Pascal's father, Étienne, had moved to Paris in 1631 to supervise his son's education and became a member of this Academy, which included René Descartes (1596-1650), and whose meetings were initially held at Mersenne's house. Étienne introduced his son to the Academy when Blaise Pascal (1623-1662)

was fourteen years old. Pierre de Carcavi, who, like Fermat, was Counsellor in the Parliament of Toulouse, was in the correspondence circle of Mersenne, and introduced Fermat into it in 1636, by motivating Étienne Pascal and Roberval to write to Fermat. In regard to what we now know as probability theory, Blaise Pascal's prime concern was the equitable division of stakes, the "problème des partis", or, in English idiom, the "Problem of Points". This is evident from his letter to the Academy entitled *Celeberrimae Mathematicos Academiae Parisiensis*, prior to his correspondence with Fermat, which alludes to work in progress entitled *Aleae geometria* (*The Mathematics of Chance*). Sometime later in that year (1654) in about early July, he wrote to Fermat almost surely about this problem. That letter has not been found, but the first surviving letter, from Fermat to Pascal, is about a simple version of the problem: if a gambler undertakes to throw a six in eight throws, but stops after the first three throws which have been unsuccessful and does not continue, what proportion of the total stake should he have? Pascal's solution is $125/1296 = (5/6)^3(1/6)$, while Fermat's is $1/6$. Fermat's would be correct if a total of 4, rather than 8, throws was originally proposed. With 8 the correct proportion (probability) would be $1 - (5/6)^5$. The second surviving letter, the famous one of 29 July, 1654 from Pascal to Fermat, discusses a more sophisticated version of the problem. In the case of two players at each of a number of trials, each has probability $1/2$ of winning the trial. It is agreed that the first player with n wins gains the total stake. The game is interrupted when player A needs a trial-wins to gain the stake, and player B needs b . How should the stake be divided? If one considers the maximum number of trials, $a + b - 1$, that the game may take to reach conclusion, the problem becomes one of "equally likely" outcomes and the calculation of the probability that A wins:

$$\sum_{r=a}^{a+b-1} \binom{a+b-1}{r} / 2^{a+b-1}$$

is made simple, although in practice the game may end earlier.

The solution by Pascal and Fermat (by different methods) even for some particular cases, was a defining epoch in probability theory.

Fermat's fame in mathematics, however, rests more in the realm of number theory. Fermat's Last Theorem provided a challenge for centuries to both professionals and amateurs.

References

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- [2] Tannery, P. and Henry, C. (eds.) (1891-1922). *Oeuvres de Fermat*. Gauthier-Villars, Paris, 5 vols.
- [3] Todhunter, I. (1865). *A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace*. Reprinted in 1949 and 1961 by Chelsea, New York.

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