Abraham DE MOIVRE
b. 26 May 1667 - d. 27 November 1754

Summary. The first textbook of a calculus of probabilities to contain a form of local central limit theorem grew out of the activities of the lonely Huguenot de Moivre who was forced up to old age to make his living by solving problems of games of chance and of annuities on lives for his clients whom he used to meet in a London coffeehouse.

Due to the fact that he was born as Abraham Moivre and educated in France for the first 18 years of his life and that he later changed his name and his nationality in order to become, as a Mr. de Moivre, an English citizen, biographical interest in him seems to be relatively restricted compared with his significance as one of the outstanding mathematicians of his time. Nearly all contemporary biographical information on de Moivre goes back to his biography of Matthew Maty, member, later secretary of the Royal Society, and a close friend of de Moivre's in old age.

Abraham Moivre stemmed from a Protestant family. His father was a Protestant surgeon from Vitry-le-François in the Champagne. From the age of five to eleven he was educated by the Catholic Péres de la doctrine Chrètienne. Then he moved to the Protestant Academy at Sedan were he mainly studied Greek. After the latter was forced to close in 1681 for its profession of faith, Moivre continued his studies at Saumur between 1682 and 1684 before joining his parents who had meanwhile moved to Paris. At that time he had studied some books on elementary mathematics and the first six books of Euclid's elements. He had even tried his hands on the small tract concerning games of chance of Christiaan Huygens (q.v.), *De ratiociniis in ludo aleae*, from 1657 without mastering it completely. In Paris he was taught mathematics by Jacques Ozanam who had made a reputation from a series of books on practical mathematics and mathematical recreations. Ozanam made his living as a private teacher of mathematics. He had extended the usual teachings of the European reckoningmasters and mathematical practitioners by what was considered as fashionable mathematics in Paris. Ozanam enjoyed a moderate financial success due to the many students he attracted. It seems plausible that young Moivre took him as a model he wanted to follow when he had to support himself. After the revocation of the Edict of Nantes in 1685 the protestant faith was not tolerated anymore in France. Hundreds of thousands of Huguenots who had refused to become catholic left France to emigrate in Protestant countries. Amongst them was Moivre who arrived in England, presumably in 1687. In England he began his occupation as a tutor in mathematics. Here he added a "de" to his name. The most plausible reason for this change is that Moivre wanted to take advantage of the prestige of a (pretended) noble birth in France in dealing with his clients many of whom were noblemen. An anecdote from this time which goes back to (de) Moivre himself tells that he cut out the pages of Newton's *Principia* of 1687 and read them while waiting for his students or walking from one to the other. True or not, the main function of this anecdote was to demonstrate that de Moivre was amongst the first true and loyal Newtonians and that as such he deserved help and protection in order to gain a better position than that of a humble tutor of mathematics. In 1692 de Moivre met with Edmond Halley and shortly afterwards with Newton. Halley cared for the publication of de Moivre's first paper on Newton's doctrine of fluxions in the *Phil. Trans.* for 1695 and saw to his election to the Royal Society in 1697. Only much later in 1735 de Moivre was elected fellow of the Berlin Academy of Sciences, and five months before his death the Paris Academy made him a foreign associate member. Looking at Newton's influence concerning university positions for mathematics and natural philosophy in England and Scotland it seemed profitable to de Moivre to engage in the solution of problems posed by the new infinitesimal calculus. In 1697 and 1698 he had published the polynomial theorem, a generalization of Newton's binomial theorem, together with application in the theory of series. This theorem was the background for a quarrel with the Scotch physician George Cheyne who had published a book in 1703 on Newton's method of fluents. De Moivre's critical remarks concerning Cheyne's book filled another book which was published in 1704. This first book of de Moivre was no success but stimulated a correspondence with Johann Bernoulli which lasted until 1714. He had tried to secure the support of Johann Bernoulli and Leibniz in order to get a professorship on the continent. De Moivre did not answer Bernoulli's last letter. It seems that de Moivre who was made a member of the commission in the Royal Society to decide in the priority dispute between Newton and Leibniz against Leibniz feared to appear disloyal to the Newtonian cause had he continued this correspondence. At any rate, the letters of Johann Bernoulli had shown to de Moivre that he lacked the time and perhaps the mathematical power to compete with a mathematician of this calibre in the new field of analysis. In addition, when the Lucasian chair in Cambridge for mathematics had been given in 1711 on Newton's recommendation to Nicholas Saunderson de Moivre had realized, that the only chance for him to survive was to continue his occupation as a tutor and consultant in mathematical affairs in the world of the coffee houses where he used to meet his clients; additional income he could draw from the publication of books and from translations. So he turned to the calculus of games of chance and probability theory which was of great interest for many of his students and where he had only a few competitors. In this respect it was easy for him to become a pioneer in a field which, apart from two episodes, he could claim for himself. In both cases he was involved in rather fierce disputes about mutual dependence with other authors in the field. De Moivre could treat the first of these, Montmort (q.v.), for personal and political reasons as an enemy without it being resented in England because Montmort was French and France had expelled de Moivre who had become a naturalized Englishman in 1705 and who had experienced the defeat of the French armies in the war of Spanish succession with grim satisfaction. Montmort had published a book on games of chance, the *Essay d'Analyse sur les Jeux de Hazard*, in 1708 and reacted to de Moivre's first publication in the field in the second edition of the *Essay* which appeared in 1713/14. The second opponent was the Englishman Thomas Simpson, who with two books from 1740 and 1742 had plagiarized de Moivre's *Doctrine of Chances* and *Annuities on Lives*.

Simpson, a former fortuneteller and weaver from Leistershire with the typical mentality of a social climber had come to London in 1736. Here he began immediately to qualify for the market of mathematically interested clients by turning out a textbook on the theory of fluxions in 1737 which was the first in a whole series of mostly very successful mathematical textbooks. De Moivre's anger and his acrimonious reaction to Simpson who had intruded into his proper domain is understandable but did not meet with general applause. In a way fortunately for de Moivre, Simpson was more successful in his efforts to get a permanent position and so dropped out from the competition for private clients in London.

Next to his clients it was Montmort who had raised de Moivre's interest in the theory of games of chance and probability. In the *Phil. Trans.* for 1711 de Moivre published a longer article on the subject which was followed by his *Doctrine of Chances*. The *Doctrine* appeared in 1718, a second edition from 1738 contained de Moivre's normal approximation to the binomial distribution which he had found in 1733. The third edition from 1756 contained as a second part the *Annuities on Lives* which had been published as a monograph for the first time in 1725.

De Moivre's preoccupation with questions concerning the conduct of a capitalist society like interest, loan, mortgage, pensions, reversions or annuities goes back at least to the 90's of the 17th century from which time a piece of paper is kept in Berlin containing de Moivre's answers to pertinent questions of a client. Halley had reconstructed from the lists of births and deaths in Breslau for each of the years 1687-1691 the demographic structure of the population of Breslau, which he assumed as stationary, in form of a life table. Halley's life table was published in the *Phil. Trans.* for 1693 together with applications to annuities on lives. Besides the formulas for the values of an annuity for a single life and for several lives he had calculated a table for the values of annuities of a single life for every fifth year of age at an interest rate of 6%. The immense calculation hindered him from doing the same for two and more lives. De Moivre solved this problem by a simplification. He replaced Halley's life table by a (piecewise) linear function. Based on such a hypothetical law of mortality and fixed rates of interest he could derive formulas for annuities of single lives and approximations for annuities of joint lives as a function of the corresponding annuities on single lives. These results were published together with the solution of problems of reversionary annuities, annuities on successive lives, tontines, and of other contracts which are depending on interest and the "probability of the duration of life" in his book *Annuities upon lives* which appeared for the first time in 1725. In the second edition of the *Doctrine of chances* part of the material contained in the *Annuities* together with new material was incorporated. After three more improved editions of the *Annuities* in 1743, 1750, and 1752 the last version of it was published in the third edition of the *Doctrine.* The *Doctrine* can be considered as the result of a competition between de Moivre on the one hand and Montmort together with Nicolaus Bernoulli (q.v.) on the other. De Moivre's representation of the solution of the then current problems tended to be more general than that of Montmort. In addition he developed a series of algebraic and analytic tools for the theory of probability like a "new algebra" for the solution of the problem of coincidences which forshadowed Boolean algebra, the method of generating functions or the theory of recurring series for the solution of difference equations. Different from Montmort, de Moivre offered in the *Doctrine* an introduction which contains the main concepts like probability, conditional probability, expectation, dependent and independent events, the multiplication rule, and the binomial distribution. De Moivre's greatest mathematical achievement is considered a form of the central limit theorem which he found in 1733 at the age of 66.

There is no doubt that de Moivre understood the importance of this special finding. From a technical point of view de Moivre understood his central limit theorem as a generalization and a sharpenning of Bernoulli's *Theorema aureum* which was later named the law of large numbers by Poisson.

Already in his commentary on Huygens (q.v.), Jakob Bernoulli (q.v.) had introduced the binomial distribution. With it he had shown that the relative frequency h_n of an event with probability p in n independent trials converges in probability to p. More precisely he had shown that for any given small positive numbers δ and ϵ then for sufficiently large n,

$$P(|h_n - p| \leq \epsilon) > 1 - \delta.$$

de Moivre, however, was interested in the precise determination of these probabilities and, by considering the ratios of binomial probabilities, he was able to show that for large n:

$$P(|h_n - p| \leq s\sqrt{(pq/n)}) \approx \sqrt{(2/\pi)} \int_0^s e^{-x^2/2} dx$$

although he did not use this representation. He calculated the value of the integral on the right hand side for $s = 1, 2, 3$. It is clear that he intuitively understood the importance of what was later called the standard deviation.

The approximation of the binomial through the normal distribution with its consequences was the culmination of the *Doctrine* from the second edition on. This book, especially the last edition of 1756,was the most complete representation of the new field in the second half of the 18th century. That this was felt by the leading mathematicians of the next generation is clear in that Lagrange and Laplace (q.v.) independently planned translations of de Moivre's *Doctrine*.

The interest of Lagrange and Laplace in de Moivre's work goes back to de Moivre's solution of the problem of the duration of play by means of what de Moivre called recurrent series and what amounts to the solution of a homogeneous linear difference equation with constant coefficients. In fact, the most effective analytical tool developed by Laplace for the calculus of probabilities, the theory of generating functions, is a consequence of Laplace's occupation with recurrent series.

However, Laplace restricted his praise of de Moivre's theory of probability to mathematical achievements like the theory of recurrent series, the central limit theorem and the hint to the generality which distinguishes the problems and their solutions chosen by de Moivre from those of his predecessors. Many other features of the *Doctrine of chances* like the interpretation of the central limit theorem concerning the relationship of probability and chance remained unmentioned by Laplace presumably because his generation did not share anymore de Moivre's views on theology and natural philosophy. De Moivre seemed to understand very different connotations of the term chance. In the first remark to his central limit theorem concerning the tossing of a coin he states, "that Chance very little disturbs the Events which in their natural Institution were designed to happen or fail, according to some determinate Law." It seems clear that chance is used in this sentence as an antithesis to law which conforms to what is called statistical law. The existence of laws of this kind is due to a "design" which again, according to contemporary convictions, at least amongst "natural" theologians, relates immediately to divine providence. Chance in contrast appears as something which obscures this design by "irregularity". Irregularity had to do with the unpredictability of the outcome in single trials and interrelated with that with deviations of a regular pattern according to which e. g. all six sides of a die should show up in some order in a series of six throws.

De Moivre did not analyse this irregularity which characterizes chance any further. However, his understanding of the concept of chance can be clarified with the help of remarks on the central limit theorem which appear only in the third edition of the *Doctrine of Chances*. He had taken the view that irregularity and unpredictability in a small number of trials, but not in the long run inherent in the concept of chance, are consistent with his repeated reference to divine design and providence. In order to understand this we must take into account that de Moivre's and Newton's creator had not abandoned his creation after its perfection but ruled it permanently in order to guarantee its existence. Therefore de Moivre's God is allwise and allpowerful; nothing happens outside his control and involvement. If one considers the status of probabilities as laws which express God's design, two conclusions are forthcoming. First, probabilities are objective properties of creation. Secondly, chance, too, as an existing property of the material world with its irregular and unpredictable aspects, is a manifestation of God's constant involvement in the course of his creation and so is objective in the sense that it is independent of the human subject and its level of information. In a way, further progress beyond such a statement is imaginable neither for de Moivre nor for Newton. Since the supreme goal of scientific enterprise is to demonstrate the existence of an agent, called God, whose constant activity permeates the whole cosmos, de Moivre with his interpretation of the interplay between law or design, which reveals the existence of God, and chance, which represents his constant activity, has reached this goal. In this sense de Moivre's chance has the same status as action at a distance in Newton's physics. Neither concept is explained by a reduction to more elementary concepts. This view differed completely from that of Jakob Bernoulli and Laplace. Both shared a credo in a completely determined world the events of which strictly follow certain laws which can be described in mathematical terms.

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