Irenée-Jules BIENAYMÉ
b. 28 August 1796 - d. 19 October 1878

Summary. Bienaymé was a Civil Servant. A disciple of Laplace, he proved the Bienaymé-Chebyshev Inequality some years before Chebyshev, and stated the Criticality Theorem of branching processes completely correctly in 1845. His work on corrrecting the use of the Duvillard life table is perhaps his greatest achievement as statistician in the public domain.

Bienaymé was born in Paris but began his secondary education at the *lycée* in Bruges, then part of the French Empire. His father held a senior administrative position in this town before moving his family back to Paris. Bienaymé entered the École Polytechnique in 1815, but this institution was closed in 1816 due to the fall of the Empire and the return of the Bourbons. With the death of his father in 1816, he entered the Ministry of Finances and rose to the rank of Inspector General in 1836. While carrying out his responsibilities as public servant, he was to become a self-made scientist, publishing first on demography and actuarial matters, and then on mathematical statistics. He was elected to the Société Philomatique de Paris in January, 1838 and was active in its affairs. His contributions to its meetings were reported in the now-obscure newspaper-journal *L'Institut, Paris*, being reprinted at the end of the year in the collections *Procès-Verbaux de la Société Philomatique de Paris-Extraits*. Most of his publications in the period 1837 to 1845 appear in this medium, and are characterized, to the frustration of the reader, by lack of mathematical proofs for assertions sometimes far ahead of their time. The most startling of his contributions occurs in this way when he gives, in 1845, a completely correct statement of the Criticality Theorem for simple branching processes, which precedes the partly correct one of F. Galton (q.v.) and H.W. Watson by over 30 years and the first subsequently correct one by over 80 (Heyde and Seneta, 1972; Bru, Jongmans and Seneta, 1992). (This theorem describes how the probability, q, of extinction of a surname depends on the average number, m, of male children per male parent. If $m \leq 1$ then $q = 1$, but if $m > 1$ then $q < 1$, and so there is a positive probability of survival of surname.) In a letter to Quetelet (q.v.) of 21 April 1846, Bienaymé confides that his everyday work and the state of his health do not permit him complete preparation of his writings for publication, and that he works seriously on applications which are of interest to both of them. His ill-health, especially his trembling hands, were to plague him to the end of his life. (Quetelet, born the same year as Bienaymé, had shortly before the letter paid a visit. Their contact was to continue, with Bienaymé's last letter to Quetelet dated September 1871).

In 1848 Bienaymé lost his job in the Ministry of Finances for political reasons associated with the changes of regime. Shortly afterwards he was asked to give some lectures on probability at the Faculté des Sciences, Paris. Again due largely to politics the Chair for probabilities was finally given to Lamé who began his course in November, 1850, and spoke thus on 26 April, 1851:

> It is my pleasure to count among my friends a savant (M. Bienaymé) who today, almost alone in France, represents the theory of probabilities, which he has cultivated with a kind of passion, and in which he has successively attacked and destroyed errors. It is to his counsels that I owe a proper understanding ...

Finally Bienaymé was reinstalled in August 1850 as "Inspecteur général des finances, chargé du service des retraites pour la vieilesse et des sociétés des secours mutuels". Although he finally resigned in April 1852, his applied statistical interests were continued in the context of the Paris Academy of Sciences (to which he was elected as *académicien libre* in July, 1852), where he was referee for 23 years for the Prize of Statistics of the Montyon Foundation, the highest French award in the area. His eminence for such a role was enhanced by the fact that he had worked hard to correct the state of affairs where upto about 1837 many insurance companies in France had used the Duvillard life table to considerable financial advantage, and the correction is deemed by some as his greatest achievement in the public domain.

The period 1851-1852 also contains Bienaymé's early contacts with J.J. Sylvester (1814-1897) and Chebyshev (q.v.), and his contribution to the enhancement of their international standing. The contact with Chebyshev was to become particularly significant.

For Bienaymé, Laplace's *Théorie analytique des probabilités* of 1812 was the guiding light, and much of his work is concerned with elaborating, generalizing and defending Laplacian positions. When the first treatise on probability in Russian (Buniakovsky's *Foundations of the Mathematical Theory of Probabilities*, clearly modelled on Laplace) appeared in 1846, one biographer of Buniakovsky claims that Bienaymé and Gauss both learned Russian in order to be able to read it. (Certainly the linguistically gifted Bienaymé knew Russian.) Bienaymé was passionate in the defence of scientific truth as he perceived it and of his friends such as Cournot (q.v.), to the extent of attacking Cauchy (q.v.) and Poisson (q.v.). J. Bertrand (1822-1900), author of *Calcul des Probabilités*, a powerful Macchiavellian figure, eventually helped 'bury' Bienaymé's reputation by unjustified criticism. Contributing to his being largely forgotten till the 1960's were the facts that Bienaymé was modest as regards his own achievements, made no great efforts to assert his priority, and was ahead of his time in mathematical statistics. He left no disciples, not being in academia; and wrote no book. However, more recently interest has revived, and on the 200th anniversary of his year of birth, at a conference in Paris, some 12 papers on his life and work were presented, in the presence of representatives of the still flourishing family Bienaymé.

It is appropriate to say something of the famous and useful Bienaymé-Chebyshev Inequality, more commonly known by Chebyshev's name alone. Both Bienaymé in 1853 and Chebyshev in 1867 proved it for sums of independent random variables. Bienaymé's proof, the simple proof which we use today, is for identically distributed random variables, treating the sample mean $\bar{X}$ in its own right as a single random variable, and is within his best known paper "Considérations à l'appui de la découverte de Laplace sur la loi de probabilité dans la méthode des moindres carrés." Chebyshev's proof is for discrete random variables and is rather more involved. Bienaymé's paper of 1853 is reprinted in 1867 in Liouville's journal immediately preceding the French version of Chebyshev's paper. The aim of both authors was a general form of the Law of Large Numbers. Eventually, in a paper presented at a conference in France and published in Liouville's journal in 1874, Chebyshev acknowledges Bienaymé's priority, and extracts from Bienaymé's approach what is the essence of the "Method of Moments". Chebyshev in 1887 used this method to give an incomplete proof of the Central Limit Theorem for sums of independent but not identically distributed summands, his final and great achievement in probability theory. This proof was then taken up and generalized by his student Markov (q.v.)

In the context of one of the polemics between Markov and P.A. Nekrasov (1853-1924) in response to a statement by Nekrasov that the idea of Bienaymé is exhausted within the works of P.L. Chebyshev, Markov says:

> The reference here to Chebyshev is misleading, and the statement of P.A. Nekrasov that the idea of Bienaymé is exhausted is contradicted by a sequence of my papers containing a generaliza-

tion of the method of Bienaymé to settings which are not even touched on in the writings of P.A. Nekrasov.

The first paper which Markov lists, published in Kazan, is that in which Markov chains first appear in his writings as a stochastically dependent sequence for which the Weak Law of Large Numbers holds. This paper was written to contradict an assertion of Nekrasov that independence was a necessary condition for this law. Thus according to Markov, Bienaymé might well be regarded as playing a role in the evolution of Markov chain theory.

The Method of Moments, however, like the Inequality, has come to be ascribed to Chebyshev.

To conclude, here is an extract from a letter written by Bienaymé on 5 April 1878, just before his own death, to E.C. Catalan (1814-1894). It is a testament, prophetic and a guide for our own times, with a touch of the old fire so evident in his controversies.

> You do not see then that everything in the world is only probabilities, or even just conjectures; and that in days to come all questions, more or less scientific, will be better understood, or even solved [in these terms] when sufficient education is given to minds capable of it by *good* teaching of probability. I don't say to all minds, as there are weak intellects, and a great number of fools ...

References

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