Marian SMOLUCHOWSKI
b. 28 May 1872 - d. 5 September 1917

Summary The investigations of the physicist Smoluchowski on the theory of Brownian movement concided with those of Einstein. The work on statistical mechanics contains probably the first manifestation of a branching process with immigration, and of statistical inference for a random process.

Smoluchowski (pronounced Smo-loo-khov-ski) was born in Vorderbrühl, near Vienna to a Polish family and died in Cracow (Polish: Kraków ; German: Krakau). The times were those of the Austro-Hungarian Monarchy of which the north-east part was Galicia (German: Galizien) with its university cities of Krakau (the old capital of Poland); and Lemberg (Ukrainian: L'viv; Polish: Lwów; Russian: L'vov), a city in Poland between World Wars 1 and 2, and now in Ukraine. Smoluchowski spent his youth in Vienna, where his father was a senior official of Emperor Franz Josef, and attended the Collegium Theresianium from 1880 to 1890. From 1890 to 1894 Smoluchowski studied physics at the University of Vienna under the direction of Josef Stefan and F. Exner. His studies were followed by a year of military service in the Austrian army, 1894/95, with his PhD in physics from Vienna University being conferred with the highest distinction in 1895. After some years of research in Paris in the laboratory of Gabriel Lippmann, in Glasgow under Lord Kelvin, in London, and in Berlin under Emil Warburg (this period being most significant for his future research), he worked at the University of Lemberg from 1899. Here he held the Chair of Mathematical Physics from 1903 to 1913. In 1913 he became Professor of Experimental Physics at the Jagiellonian University in Cracow. He died of dysentery (doubtless due to wartime conditions) in the summer of 1917 at the age of 45 before he could take up his duties as rector of this university. His most important work was done during the period in L'vov, and is heavily probabilistic and statistical. It is regrettable, as the later Nobel laureate Chandrasekhar (1943) makes clear several times, that Smoluchowski's lucid contributions to the theory of Brownian motion, molecular fluctuations and laws of thermodynamics, have beem neglected. One might add to this list the theory of branching processes, where his contributions have long occupied a neglected position similar to that of I.J. Bienaymé (q.v.).

From about 1900 Smoluchowski worked on Brownian movement, and obtained a theory starting from the standpoint of examining the effects of successive collisions between a Brownian particle and successive molecules. Einstein started from general relations of statistical physics, and presented a theory of the Brownian movement in papers of 1905 and 1906. Smoluchowski had waited to use experimental data for verification of his own theory, which was little different, but in view of Einstein's first paper, published his findings in 1906. When Einstein was awarded the Nobel Prize in Physics in 1921, his work on Brownian motion is cited, before the discussion of the main reason for the prize : the discovery of the law of photoelectric effect.

Smoluchowski's theory of density fluctuations is within the context of the Brownian motion work, as the title of the 1914 paper testifies. Chandrasekhar (1943) described it as "one of the most outstanding achievements in molecular physics". Smoluchowski here modelled the fluctuation in the number of particles contained in a geometrically well-defined small element of volume, v, within a much larger volume of solution containing particles exhibiting random motion, the system being in equilibrium. Observations $X_1, X_2, \cdots$ on the number of particles in v are made at points of time at equal intervals, τ, apart. During such a time interval some particles present in v at its beginning will have left it (have "died") by its end while some external to v at the beginning of the interval may be present within v at its end (have "immigrated"). The probability P that a particle present at the beginning will be external at the end depends on the precise circumstances of the problem. An explicit *theoretical* solution was obtained by Smoluchowski (1914) when the motions are governed by the laws of Brownian movement, in terms of the various physical parameters. Since the system is in equilibrium, and v is small, the probability distribution describing each X_n is Poisson, with mean/variance parameter μ, say : $Pr(X_n = x) = e^{-\mu}\mu^x/x!,\ x = 0, 1, 2, \cdots$. Both P and μ can be estimated *statistically* from the numerical observations on $X_1, X_2, \cdots$. A comparison of the predictions of the theory of colloid statistics with the data is therefore made possible, and was in fact carried out on the data of Th. Svedberg by Smoluchowski himself. In regard to the statistical estimation procedure, Smoluchowski derived the elegant probabilistic expression $E((X_{n+1} - X_n)^2) = 2\mu P$. Its left-hand side was estimated, using observations $X_1, \cdots, X_{N+1}$, naturally by

$$\sum_{i=1}^{N}(X_{i+1} - X_i)^2/N$$

while μ was estimated by $\hat{\mu} = \sum_{i=1}^{N}(X_i - \bar{X})^2/N$ where $\bar{X} = \sum_{i=1}^{N} X_i/N$, thus giving $\hat{P}$. Shortly after, it became common to use the estimator $\hat{\mu} = \bar{X}$ for μ (Fürth, 1918).

The striking advance here on earlier fluctuation theory was the introduction of the *probability after-effect ("Wahrscheinlichkeitsnachwirkung")* P , which is a manifestation of the assumed Markovian ("given the present, the future is independent of the past") structure of the assumed model, and is of great significance in other physical contexts. Indeed the random process $\{X_t\}$ used as a model by Smoluchowski is a Bernoulli-Poisson branching process with immigration, apparently the first time a branching process with immigration of any kind occurs in the literature. Branching processes with immigration are examples of Markov (q.v.) chains, which arose, at least as a general concept, circa 1906. This Markovian aspect of Smoluchowski's work was overlooked in German-language literature. The transition probabilities of the Bernoulli-Poisson Markov chain, which are of simple explicit kind, were however already present, both as a physical idea and in explicit form in Smoluchowski (1914).

It is worth reflecting on the fact that at the time of Smoluchowski's work, statistical estimation theory, even for independent samples, was in its infancy, and yet he used totally appropriate estimators in the more complex setting of a Markov chain.

Sommerfeld (1917) in his obituary lists 53 works by Smoluchowski over the period 1893-1917, mainly in the official language of the Austro-Hungarian Monarchy, German, but some in French and English; and mentions the existence of some 36 further works in Polish.

Smoluchowski married Zofia Baraniecka, the daughter of a mathematics professor at the Jagiellonian University, in 1901 and lived contentedly in Lemberg, taking an active part in the life of the university and the community, until the move to Cracow. He was fond of mountaineering, skiing and music in the company of his close friend Hasenörhl, who was killed in World War 1, in 1915. Smoluchowski's wife outlived him by some 40 years, and died in the U.S.A. as did his son, Roman Smoluchowski (1910 - 1996), a notable physicist. There is also family in Poland.

Sommerfeld(1917) refers to him as: Marian Ritter von Smolan-Smoluchowski ("Ritter" translates into "knight".)

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