Adrien-Marie LEGENDRE[1]
b. 18 September 1752 – d. 9 January 1833

Summary. In 1805, Legendre published the first description of the method of least squares as an algebraic fitting procedure. It was subsequently justified on statistical grounds by Gauss and Laplace.

Adrien-Marie Legendre was born in Paris (France). He was the son of well-to-do parents and could afford to devote himself full-time to scientific research until the rigours of the French Revolution dissipated the family fortunes and made it necessary for him to work for a living as a minor official in a variety of administrative posts.

The French Revolution began in May 1789, King Louis XVI was executed in January 1793 and Queen Marie-Antoinette in October 1793. the subsequent Reign of Terror hastened the deaths of many scientists including those of Antoine Lavoisier and the Marquis de Condorcet (q.v.). It is fortunate for the development of science in France that Fourier, Laplace (q.v.), and Legendre amongst others were able to escape with their lives.

Following the completion of the Revolution in France, the French government, latterly under the control of Napoléon Bonaparte, fought a series of wars which drew in most of the countries of Europe. Thus it was in the year of the battles of Austerlitz and Jena, that Legendre, then 52 years old, published his monograph outlining the fundamental results on the method of least squares.

Legendre was educated at the *Collège Mazarin*, graduating in 1770. Between 1775 and 1780 he taught mathematics at the *École Militaire*.

The geodetic work published by Legendre during this period was so important that it merited four chapters in Todhunter's *History of Mathematical Theories of Attraction and the Figure of the Earth (1873)*. Thus, when in 1787, the *Académie des Sciences* was asked to nominate three members of a commission to connect the maps of Britain and France by determining the relationship between the meridians of Greenwich and Paris, it chose Legendre, Méchain, and J.D.Cassini. In 1791 the *Académie* again nominated the same individuals as its members of a commission to determine the length of the

[1] Recent research has shown that the portrait employed in the hard copy edition of Statisticians of the Centuries and elsewhere is actually that of the politician Louis Legendre (1752–1797).

mètre which was to form the basis of a new decimal system of mensuration. However, Legendre resigned from the second commission in March 1792.

In 1799 Laplace became Minister of the Interior and Legendre succeeded him as the examiner in mathematics of students assigned to the artillery corps. In 1815 Legendre voluntarily resigned this position on half pay but was subsequently stripped of his pension when he refused to vote for the official candidate in an election to a seat in the *Institut de France*.

Legendre died in Paris in 1833.

In the second half of the eighteenth century astronomers, geodesists, and other practical scientists were obliged to perfect their own methods for solving systems of equations in which there were more observations than unknowns. From antiquity until 1750 the only possible solution to this problem was to arrange that there should be as many equations as there were unknowns. In 1750 Tobias Mayer suggested that the unknowns should be determined by setting certain sums of equations equal to zero; in 1760 Boscovich (q.v.) proposed that the unknowns should be determined by minimising the sum of the absolute errors subject to an adding-up constraint; and in 1786 Laplace suggested that the unknowns should be determined by minimising the largest absolute error. Legendre's method of least squares clearly represents a further contribution in this practical tradition as his argument is entirely algebraic and has no statistical content.

Legendre's discussion of the method of least squares is to be found in the first four pages (pp.72-75) of a nine-page appendix attached to his 1805 work on the determination of the orbits of comets. The remaining five pages of this appendix are concerned with an application of the method of least squares to the determination of the ellipticity of the Earth, and hence the length of the *mètre*.

By contrast with the Boscovich and minimax procedures which, in practice, seem to have been restricted to the case of two unknowns, the method proposed by Legendre could be applied to any number of linear equations in any lesser number of unknowns. Instead of choosing values for the unknowns to minimise the largest absolute error or to minimise the sum of the absolute errors, he chose these values to minimise the sum of the squared errors. Legendre wrote down the first order conditions for the minimisation of this function. He then noted that each of these conditions may be obtained by multiplying all the terms in each of the equations by the coefficient of one of the unknowns and summing the result. He was thus able to establish that there were the same number of equations as there were unknowns, and hence that the first order conditions could be solved for the unknowns by a standard procedure which he did not describe.

Legendre asserted that the solution found by this procedure corresponds to a minimum of the sum of squares function . But he did not prove this result except in the special case when there is an exact fit and the least squares values of the unknowns are associated with a set of zero errors. Further, if the least squares solution produces any large errors then Legendre suggested that the corresponding observations should be discarded and new least squares values computed by deleting the corresponding terms from the calculations. This statement presumably represents an extension of current practice to the method of least squares. The problem of deciding when an error is large enough to warrant deletion was not discussed. Legendre gave very little justification for choosing the sum of the squared errors as his optimality criterion other than its computational simplicity. However he did note that it yields the arithmetic mean when there are a set of direct observations on a single unknown quantity. Subsequently, in the nonstatistical section of his *Theoria Motus Corporum Coelestium (1809)*, Gauss (q.v.) essentially reproduced Legendre's deliberations but offering the sum of the fourth powers, the sum of the sixth powers, etc as possible alternatives to the sum of the squared errors. Of these criteria, the simplest is indeed the sum of the squared errors, which Gauss says he had used in practical calculations since 1795, much to the annoyance of Legendre who published a venomous response in 1820.

The method of least squares would seem to have been hanging in the air at this time as there are several claimants for the privilege of having first invented it including Gauss, Cotes, Simpson, and Huber. Most of these claims can be rejected as having been made with the benefit of hindsight and in ignorance of the difficulty that eighteenth century mathematicians would have experienced in conceiving of minimising a sum of *squared* errors rather than a sum of absolute errors. However, the claim for priority made by Gauss at the time would seem to have been substantiated in the opinion of Laplace, Plackett (1972), Stigler (1977, 1981, 1986), and other authorities, but Legendre retains the priority of publication, for what that is worth. Further, Gauss's practical fitting procedure was designed for use with nonlinear problems and its precise nature is still open to question, again see Stigler (1981) for details.

As noted above, Legendre's derivation of the method of least squares was entirely algebraic; statistical justifications for this fitting procedure were subsequently provided by Gauss, Laplace, Cauchy (q.v.), and Thiele (q.v.) amongst others.

Legendre's system of orthogonal polynomials have also found applications in statistics.

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