**Pierre-Simon Marquis de LAPLACE**
b. 23 March 1749 - d. 5 March 1827

**Summary.** Pierre-Simon Laplace was the most prominent exponent of 19th century probability theory. His major probabilistic work, the *Théorie analytique des probabilités* considerably influenced the development of mathematical probability and statistics right to the beginning of the 20th century.

## Introduction

Pierre-Simon Laplace was born in Beaumont-en-Auge (Normandy). His parents, Pierre and Marie-Anne, née Sochon, lived in comfortable bourgeouis circumstances. Laplace's scientific career evolved in a period of political upheaval, but it continued to flourish in all political systems (1789 French Revolution, 1799 Napoleon's seizure of power, 1815 reestablishment of the monarchy). Originally destinated to become a priest, Laplace soon discovered his mathematical talents. Supported by d'Alembert (q.v.), he obtained a professorship at the *École Militaire* in 1771. In 1773 he was admitted to the *Académie des Sciences de Paris* of which he became one of the leading members in the 1780's. After the revolution, Laplace played a decisive role in the commission of weights and measures aiming at the introduction of the metric system. Around 1795 he became very influential in the organization and teaching of the newly established *École Polytechnique* and *École Normale*. Laplace served only 6 weeks as Napoleon's unfortunate Minister of the Interior in 1799, thereafter he was honorably transferred to the Senate of which he became Chancellor in 1803. Louis XVIII, too, had the highest esteem for Laplace. In 1816 he was admitted to the *Académie Française* and in 1817 he was raised to the rank of a Marquis. Laplace died on 5 March 1827 in Paris, having pursued his scientific research very actively almost to the end of his life.

## Scientific Work in General

Laplace contributed numerous articles to optics, acoustics, heat and capillarity. In his purely mathematical papers, he dealt mainly with difference and differential equations. The focal points of Laplace's scientific activities were theoretical astronomy and probability theory, which contained, according to Laplace's approach, also those parts today considered as "statistics". From Laplace's point of view, a differentiation between "probability theory" and "statistics" would not have been appropriate. In 1796 the popular *Exposition du système du monde* appeared and between 1799 and 1805 the first 4 volumes of the *Traité de mécanique céleste*, the fifth volume of which was published by Laplace in 1825 ((*Œuvres I-V*). In his work on physics and astronomy, which made him the leading figure within French natural sciences, Laplace became the most prominent propagator of the idea that, in principle, each condition in the world could, according to the pattern of determining position and velocity of celestial bodies, be precalculated. Laplace put this strictly deterministic point of view in concrete form by his proverbial "Laplacean demon". The *Théorie analytique des probabilités* (Ist ed. 1812, 2nd ed. 1814, with an extensive introduction and a chapter on the probability of testimonies, 3rd ed. 1820 with supplements = (Œuvres VII) was the sum of Laplace's probabilistic work since 1774. The introduction which was added to the *Théorie analytique* from the 2nd ed. was also published separately between 1814 and 1825 in 5 editions under the title *Essai philosophique sur les probabilités.*

**Philosophy of Probability**

Laplace held the view that man, in contrast to the "demon", was capable of achieving only partial knowledge about the causes and laws which regulate the processes of the cosmos, but he maintained that probability theory was a means to overcome this defiency. In accordance with this concept, Laplace put special emphasis on subjective probabilities depending on the degree of information, but the frequentistic notion of probability is also used in Laplace's work in many places. Laplace was convinced of the universal applicability of probability calculus and he summarized this opinion, which he shared with all probabilists of the enlightenment, by the words: "Probability is basically good sense, reduced to a calculus."

**Analytic Methods of Probability Theory**

Laplace considered his form of probability theory, as described in the *Théorie analytique*, important not only because of its universal applicability but also because of its innovative analytical methods. Actually, no probabilist before Laplace was able to offer results which could have been compared with the analytical content of the ones presented by Laplace. Consequently, the *Théorie analytique* was divided into two books, the first dealing exclusively with the analytical apparatus, in particular with the application of generating functions to difference equations and techniques for calculating and approximating definite integrals. Laplace already used, albeit in a still rudimentary form, characteristic functions for the representation of the probabilities of sums of independent random variables. In its emphasis on the analytical importance of probabilistic problems, especially in the context of the "approximation of formula functions of large numbers," Laplace's work goes beyond the contemporary view which almost exclusively considered aspects of practical applicability.

**Bayesian Methods and Population Statistics**

A considerable part of Laplace's contributions, which would be considered today as belonging to "mathematical statistics", was based on inverse probabilities. By this method, the a posteriori probability of a certain hypothesis could be calculated from the results of random experiments, usually under the tacit assumption of an a priori equiprobability of all possible hypotheses. We are not sure about whether Laplace began his inquiries with or without a knowledge of Bayes' (q.v.) fundamental treatise (1764) on this issue. By the aid of suitable approximations to his resultant formulas - a problem which Bayes had failed to solve - Laplace showed in several papers, published between 1774 and 1786, that, on the basis of the existing data, the probability of a boy's birth is, almost infallibly, greater than 1/2; that the birth rate for boys in London is in all likelihood greater than in Paris, and so on. One can suppose that reports on death rates in French hospitals, published by the *Académie des Sciences*, were also based on similar Laplacian calculations. Together with Condorcet (q.v.) and Séjour, Laplace was a member of the commission of the *Académie des Sciences* which organized, in the 1780's, the publication of several papers concerning population statistics in all parts of France, based chiefly on data sampled by La Michodiére. In these statistical investigations the idea of a micro census, as already used by Graunt (q.v.) was pursued: The ratio between the number of persons and the number of births per year within a suitable selection of population must be approximately equal to the ratio between the total number of persons and the total number of births per year. By a Bayesian approach, Laplace calculated the probability of the deviation of the estimated value for the total number of persons from its actual value, if the estimation was obtained by equating both ratios. Between 1799 and 1802 a micro census was organized for the whole of France according to "Laplace's method" ((*Œuvres* VII, 398-401). Laplace's interest in population statistics, however, was apparently less motivated by social or political concerns, than by the scientific aim of making evident that the social world can basically be approached by the same probabilistic methods as the physical.

**Central Limit Theorem, Asymptotic Error Theory**

Laplace's main probabilistic result was a fairly general central limit theorem, which was obtained around 1810. This theorem assures an approximate normal distribution for practically all sums of independent random variables in nature and society, if only the number of the summands is large. This result, although it was nowhere explicitely formulated, but in each case deduced in the context of its special applications, was to become a leitmotif of Laplace's *Théorie analytique*. On the basis of approximate normal distributions of linear combinations of errors of observation, Laplace succeeded in showing that the method of least squares is, according to various criteria, asymptotically "most advantageous" for estimating the parameters of linear models which occur in the context of astronomical or geodetic observations. Thus, he presented basic ideas of asymptotic statistics within the scope of error theory. Error calculus also served Laplace as a pattern for the determination of natural regularities hidden by irregular fluctuations, such as weather conditions. An important example was the "constant" difference of the air pressures in the morning and in the afternoon. For an assessment of whether assumed regularities actually existed, Laplace's central limit theorem allowed a reasoning similar to the one used in modern tests of significance, provided that the test statistics were sums of a large number of independent random variables. On the basis of central limit theorems, Laplace arrived at a probabilistic discussion of mean errors of observation, mean gains of gambles or mean durations of life, and in this context one can find statements which today would be called weak laws of large numbers.

**Probability and Moral Sciences**

In continuation of the work of Condorcet, Laplace reflected on the field of erroneous human decisions, such as testimonies or verdicts, within the framework of urn models. In view of the oversimplified models Laplace expressed certain reservations, but he emphasized at the same time the advantages of probabilistic "estimations". In the first supplement of his *Théorie analytique*, Laplace calculated the a posteriori probability that the defendent is actually guilty, if $n$, votes have been cast against him, under the double presupposition that among $n$ members of a jury the same probability $x$ of a correct decision in the case of guilt can be assigned to all of them, and that all values $x$ are a priority uniformly distributed between 1/2 and 1. On the basis of these calculations, Laplace gave recommendations for the composition of, and the majority within, Juries, which he also published in a pamphlet in 1816 ((*Œuvres* VII, 529f.) and repeated in a speech at the *Chambre des Pairs* (*Œuvres* XIV, 379-381) in 1821. Laplace's arguments were repeatedly brought forward in the frequent discussions of the 1820's and early 1830's about jury systems in France. At the same time, however, probabilistic reasoning within moral sciences was increasingly criticized by philosophers and mathematicians. This fell especially upon Poisson, who amplified Laplace's inquiries on moral questions by the use of a great deal of statistical data. Following Poisson (q.v.), there has been little active research in this part of classical probability theory.

**Impact of Laplacian Probability**

To the end of the 19th century, Laplace's *Théorie analytique* remained the most influential book of mathematical probability theory, which was considered less a part of mathematics in the narrower sense, but a discipline of "mathesis mixta". Reduced as it was by a major field of application of the classical theory, the moral sciences, and augmented only by problems which could be mastered within the framework of simple stochastic techniques, such as the kinetic theory of gases, hardly any probabilistic concepts were put forward which were new with regard to Laplace's.

In the field of statistics, Laplace had mainly presented theoretical concepts in a rather unsystematic way in his *Théorie analytique*. His analytical deductions were written in a very difficult style, and his mode of reasoning within error theory became far less popular in comparison with Gauss' (q.v.), which was easier to understand and to apply. The general relevance for statistics of Laplacian error theory was appreciated only by the end of the 19th century. However, it influenced the further development of a largely analytically oriented probability theory; limit distributions of sums of independent random variables became a basis of modern probability theory.

In addition, some basic ideas, chiefly disseminated by Laplace in a verbal form in his *Essai philosophique*, decisively influenced 19th century statistics in producing the expectancy, that all random fluctuations, in nature and society, could be treated correspondingly to the pattern of errors of observations. This concept, together with Laplace's frequent approximations by normal distributions which, however, he did not investigate as statistical objects in their own right, paved the way for the later "Quetelism".

**Literature**

The ( *Œuvres complètes de Laplace*, which appeared between 1878 and 1912 (Paris, Gauthier-Villars) are not really complete. The most comprehensive scientific biography is C.C. Gillispie, *Pierre-Simon Laplace, 1749-1827, A Life in Exact Science* Princeton University Press, Princeton, 1997.

S.M. Stigler's book *The History of Statistics*, Belknap, Cambridge, MA, 1986 contains a detailed historical discussion of Laplace's most relevant stochastic contributions.

Descriptions of mathematical details can be found in O.B. Sheynin, P.S. Laplace's Work on Probability, *Archive for History of Exact Sciences*, **16** (1976), 137-187, idem, Laplace's Theory of Error, *Archive for History of Exact Sciences*, **17**, (1977), 1-61, and concerning Bayesian Methods in A.I. Dale's book *A History of Inverse Probability*, Springer, New York, 1991.

The intellectual background of Laplace's probabilistic work together with his treatment of moral questions is discussed in L. Daston's book *Classical Probability in the Enlightenment*, Princeton University Press, Princeton, 1988.

I. Schneider, (1987). Laplace and Thereafter. In *The Probabilistic Revolution*, Vol. 1, ed. by L. Krüger *et al.*, MIT-Press, Cambridge, MA, pp. 191-214, gives a history of the impact of Laplacian probability theory in the 19th century.

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