Félix POLLACZEK
b. 1 December 1892 - d. 29 April 1981

Summary. Pollaczek was a pioneer in mathematical modelling of queueing phenomena. He was a fine scientist and made fundamental contributions to queueing theory.

Félix Pollaczek was born in 1892 in Vienna and passed away in at his home in Boulange-Billancourt, France. In Vienna he attended the "Akademie Gymnasium" (latin grammar school). In 1910 he enrolled as a student in electrical engineering at the Technological University Vienna. During the first World War he served in the Austrian army and reached the rank of lieutenant. In 1921 he married Hilda Geiringer. His only child Magda, a daughter, was born in 1922. They soon separated. In 1944 Hilda became the wife of von Mises (q.v.) [1]. Felix remarried in 1934 to Vera Jacobowitz. It was a happy marriage, and Vera was a great support for him.

In 1916 Pollaczek wrote his first paper. It was on Fermat's (q.v.) Last Theorem and published in the 1917 proceedings of the Imperial Vienna Academy of Sciences. In 1918 he resumed his studes and obtained his master's degree in 1920 at the Technological University of Brno, Czechoslovakia. He then moved to Berlin and obtained his doctor's degree in 1922 from the University of Berlin with a thesis on number theory under supervision of I. Shur.

In 1921 he was employed by the A.E.G., a large German electrical company. In 1923 he joined the research institute of the German Post Office. The Nazi rules forced him, being a Jew, to leave in 1933. He went to Paris. Vaulot, also a pioneer in telephone congestion theory, got him a consultant position with the "Société d'Études pour Liasons Téléphoniques et Télégraphiques". Visa problems caused him to leave in 1936. He and Vera returned to Brno, where his mother's family lived. Khintchine invited him to Russia in 1937. He spent three months there, and visited several universities; a professorship was offered to him by the University of Tiflis. Difficulties with a visa and the unattractive political climate in the Soviet Union made him renounce this position. When the Germans invaded Czechoslovakia in 1938, a last minute escape brought Félix and Vera to Paris again. During 1939/1940 he was consulting with "Center National de la Recherche Scientique" (CNRS) as "Maître de Recherches". The war time was hard for them, but they succeeded in escaping the Nazi persecution of the Jews, see [3]. After the liberation of France in 1944 Pollaczek again became affiliated with CNRS. Although already in his fifties, Pollaczek was unbroken. He resumed his research, published, visited congresses and accepted invitations to lecture. His economic position was rather difficult, but in the mid fifties a pension from his former German employer made the life of Félix and Vera more relaxed. In 1947 they obtained French nationality.

Pollaczek's publications are listed in [4]. The list, as composed by him, is divided into five groups: Number Theory, Mathematical Analysis, Mathematical Physics, Rational Mechanics and Probability Theory. Number Theory contains three papers, the last one dates from 1929. Eleven papers on Mathematical Analysis, all published after 1944, for the greater part they concern orthogonal polynomials, see also Pollaczek polynomials in [11]. The fourteen ones on Mathematical Physics concern mainly electro-magnetic problems arising in the design of transmission lines and cables. Almost all of them date from Pollaczek's time with the German Post Office. The paper in Mechanics is on wave propagation. Those listed under Probability Theory form the greater part of his publications among them two monographs, one on the single server waiting time model [9] the other on the many server model [10].

Pollaczek is best known for his mean waiting time formula for the stable M/G/1 queueing model derived in 1930 [5]. This famous formula is known as the Pollaczek-Khintchine formula. Khintchine [6] derived it independently a few years later.

The paper [5] of Pollaczek was the second study in queueing literature of a queueing model with an arbitrary service time distribution. The first one stems from Vaulot [7]. In [7] it is shown that the stationary distribution of the number of busy servers for the classical Erlang loss model depends only on the first moment of the service time distribution and not on any other characteristic of this distribution; the so called "insensitivity property". Forty years later insensitivity phenomena became an important research subject in the performance analysis of queueing networks. In his paper [7] Vaulot introduced the so-called method of "the supplementary variable"; it became an important tool in analytic Queueing Theory. Former modelling of queueing situations used negative exponential distributions for the description of interarrival- and service time distributions. With this assumption an analytic approach, presently known as the birth-and-death technique, could be applied. This technique has been extensively used by E.C. Molina from the Bell System Laboratories (see [8] for details.

The analysis of the one- and many server waiting time model with unspecified interarrival- and service time distributions, the GI/G/s model, became the main theme of Pollazcek's research. His monograph [9] discussed the GI/G/1 model, and it is a masterpiece of classical hard analysis. The starting point is the relation

$$W_{n+1} = max(0, w_n + \tau_n - \sigma_{n+1}) \ .$$

Here w_n is the waiting time of the nth arriving customer, τ_n his service time and σ_{n+1} the time between the arrivals of the nth and $(n+1)$th customer. The equation is a specimen of relations encountered in waiting time- and storage models. The relaion is transformed into a singular integral equation for the generating function

$$\Phi_\omega(\tau, \rho) := \sum_{n=1}^{\infty} \tau^n \mathrm{E}\{e^{-\rho w_n} | w_1 = \omega\}, \ |\tau| < 1, \ \mathrm{Re}\, \rho \geq 0 \ ,$$

of the sequence of Laplace-Stieltjes transforms of the distributions of w_n. The explicit solution of the integral equation for $\Phi(\tau, \ \rho)$ by means of an integral expression was a remarkable achievement. The only assumption introduced is that of the regularity of the L.S.-transform $\beta(\rho)$ of the service time distribution $B(t)$ at $\rho = 0$. It implies that $B(t)$ should not have too heavy a tail. The assumption, however, can be removed and the result can be used for heavy-tailed distributions $B(.)$ which arise in current modelling.

The technique developed by Pollaczek also leads to explicit results for busy period and idle time distributions, queue length distributions and the covariance of the sequence w_n for $n \to \infty$ is another important result. Pollaczek's approach became a powerful technique for the analysis of a large class of models in Queueing Theory, Storage, Risk and Dam Theory. Around 1960 Fluctuation Theory was developed. It soon appeared that the probabilistic structure of the GI/G/1 model was closely related to those studied in Fluctuation Theory, and that many of Pollazcek's results could be rephrased in terms of stochastic characteristics of partial sums of independent random variables.

Pollaczek seems to have been little interested in the stochastic structure of the processes which he investigated. His goal was to provide the designers of telephone systems with sharp tools to judge the system performance which was measured in terms of congestion- and delay probabilities, average waiting times and queue lengths. His approach was the classical one of Applied

Mathematics. Make a model which incorporates all the essential features of the actual engineering problem. Once an adequate model had been set-up its analysis is a mathematical problem, which was solved only if the solution is amenable to numerical evaluation. It is a point he mentioned in several conversations. He considered the analysis of the GI/G/1 model as trivial. The GI/G/s, $s > 1$, is the really hard one. He was never fully satisfied with his analysis of it, see the monograph [10], and still worked on it in the late seventies. Numerical accessibility was the problem, but rather recently the tools became available.

Pollaczek was a sharp and original mathematician, whose work strongly influenced analytical Queueing Theory. In the eighties it turned out that his approach fits well into the class of Riemann-Hilbert Boundary Value Problems, a chapter in the theory of integral equations. In this context the GI/G/1 is indeed almost trivial. However, not so the many server model.

Pollaczek was a kind and modest man, maybe too modest; at conferences he usually remained in the background. He was a learned man, who fostered scientific standards and integrity. In 1977 he was awarded the John van Neumann Theory Prize by the Operations Research Society of America.

The reference [3] contains a detailed biography of Pollaczek composed by Schreiber and Le Gall. The details about his course of life are borrowed from their paper.

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