Daniel BERNOULLI
b. 8 February 1700 - d. 17 March 1782

Summary. Daniel Bernoulli, well known as a mathematician, provided the earliest mathematical model describing an infectious disease. In 1760 he modelled the spread of smallpox, which was prevalent at the time, and argued the advantages of variolation.

One of the purposes of modelling the spread of an infectious disease mathematically is to provide a framework within which predictions can be made about its progress within a population susceptible to infection. Perhaps the first such model for an infectious disease was that for smallpox due to Daniel Bernoulli.

Daniel, who was born in Groningen in 1700, was a member of the famous Bernoulli mathematical family, the second son of Johann I (Jean) Bernoulli, brother of the older and more famous Jakob I (Jacques). He studied mathematics under his father and medicine at Heidelberg receiving his M.D. in 1721. In 1724 he published "Exercitationes Mathematicae", solving Riccati's equation, and in 1725, he was appointed to the Academy of Sciences of St. Petersburg, which had just been founded by Peter the Great. He remained there until 1732.

After Peter's death in 1725, there followed a period of unrest until the imperial succession was settled. Bernoulli returned to Basel in 1733, to accept a post in Anatomy and Botany; this was followed in 1743 by one in Physiology and in 1750 by another in Physics. His interests remained very diverse, and he pursued his research into mathematics (including probability), medicine, physics, botany, anatomy and philosophy, writing several treatises on these subjects. In 1734 he published his "Memoire sur l'Inclinaison des Orbites Planetaires", followed in 1738 by his "Traite d'Hydrodynamique", and a paper on the measurement of risk (1738). In 1740 his "Traite sur les Marees" appeared, and shortly after, his work on vibrating strings and rods. He was involved in quarrels with his friend Euler on the mathematical theory of the vibrating string, and with D'Alembert (q.v.) on his work on risk and the merits of variolation.

Basel was very close to Alsace and the French frontier, and while the Swiss cantons maintained their independence, intellectual and particularly scientific life was centered on France and its Royal French Academy of Sciences in Paris. Between 1725 and 1749, Bernoulli won 10 Prizes from this

Academy, including the Prize of Honour for his clepsydra (water clock) to measure time at sea.

For most of Bernoulli's mature life, the intellectual climate was dominated by the "philosophes" who believed in rationalism and enlightenment; their views were presented in the *Encyclopedie* of Diderot (1713-1784), for which D'Alembert wrote the Introduction. Diderot himself was the author of an article on probability and inoculation commenting on D'Alembert's critiques of Bernoulli's papers on these topics. Although France suffered from its defeat in the Seven Years War (1756-1763), its population had grown rapidly in the later half of the 18th century, as famines and epidemics became rarer; it reached some 26 million by 1789. The size and health of the population was of vital concern in the recruitment of soldiers for the king's army.

Bernoulli was interested in the ravages caused by smallpox, which was prevalent at the time. He devised a simple deterministic model to relate the size of a cohort $w(t)$ of individuals at time t after birth, with the number of susceptibles $x(t)$ among them who had not been infected with smallpox. To make his equations easier, he assumed that individuals infected by smallpox died immediately, or recovered immediately and became immunes $z(t)$, so that $w(t) = x(t) + z(t)$. By setting up and solving two ordinary differential equations he obtained the relation

$$x(t) = w(t)/[(1 - a)e^{bt} + a]$$

where b is the rate of catching smallpox and a the proportion of those infected with smallpox who die instantaneously. Setting $a = 1/8$ and $b = 1/8$, and using Halley's (1693) life tables he was able to estimate the numbers of susceptibles in a cohort subject to smallpox at time $t > 0$, as well as the increased expectation of life if smallpox were eliminated. He calculated that this expectation would increase from 26 years 7 months to 29 years 9 months. If smallpox could be eliminated, then by age 26, the population would be some 14% larger. He used his results to argue the advantages of variolation; in an earlier popular article (1760), he wrote "The two great motives for inoculation are humanity and the interest of the State". It should be mentioned that the Bernoulli family in Basel had several of its younger members variolated.

Variolation, or the inoculation of susceptibles by live virus from patients with a mild form of the disease, was becoming popular in Europe at that time. The practice had originated in China as early as 1,000 AD; Chinese doctors had ground the scabs of dried smallpox pustules into powder, this being inhaled by individuals wishing to acquire immunity. The custom had migrated to India and Turkey in a modified form involving inoculation with live virus from smallpox pustules. In the early 18th century, travellers to Turkey had reported on the success of variolation, and some doctors in Europe began to practise it. Unfortunately, even virus from a patient with a mild form of smallpox did not guarantee a mild attack in the new host.

In a paper presented to the Royal French Academy of Sciences in Paris in 1760, Bernoulli set out to present the advantages of variolation. He became involved in a quarrel with D'Alembert on the interpretation of the relative risks of death from smallpox and variolation; D'Alembert maintained that the real risks of the latter were 17 times greater than those of smallpox itself. However, he eventually moderated his objections, and the paper was finally published in 1766. The potential benefits of variolation became irrelevant within 3 decades, as Jenner demonstrated the safety of the cowpox vaccine against smallpox in England in 1796.

Among his many mathematical works, Daniel Bernoulli also made important contributions to probability theory, for example to what became known as the St. Petersburg paradox, to which his cousin Nicholas Bernoulli (q.v.) also contributed usefully. Sheynin (1972) refers to 8 memoirs on probability published between 1738 and 1780, also briefly described in Todhunter (1865). Possibly the most interesting are the memoirs on the normal law, the measurement of risk (1738), the theory of errors, and various urn problems. In his "Traite d'Hydrodynamique", Bernoulli studied the effect of pressure and temperature on gases; assuming that a gas consisted of small particles, he treated the problem by the probabilistic methods of Pascal (q.v.) and Fermat (q.v.). This was the first attempt to develop a kinetic theory of gases, a feat later achieved by Maxwell and Boltzmann in the nineteenth century.

Bernoulli died in Basel in 1782, laden with honours, a polymath whose works on mathematics, probability and infectious disease modelling were recognized for their originality, depth and practical applicability.

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