Gustav Theodor FECHNER
b. 19 April 1801 - d. 18 November 1887

Summary. Physicist, psychologist and philosopher, Fechner is noted for the introduction of quantitative methods into psychology. He also developed a 'theory of collectives' which is built on the frequency interpretation of probability.

Gustav Theodor Fechner was born in Gross Särchen near Muskau, Lusatia (Germany), into the family of a protestant minister. He studied medicine, but became disenchanted with the subject and never wanted to practice it. Instead, he devoted himself to experiments in physics in the fashion of the *physique expérimentale* of the leading French physicists. His translations and revisions of French text books and treatises were the chief channel for the reception of French mathematical science into Germany at this time, and the reform of German physics that resulted from it. His experimental researches, especially in electricity theory, eventually earned him a chair in physics at the University of Leipzig (in Saxony, Germany) where he remained for the rest of his life.

Fechner's orientation towards the most advanced physics of his day was supplemented, however, by a strong commitment to idealist and romantic *Naturphilosophie* which was primarily directed against Cartesian dualism of mind and body and eighteenth-century French materialism. Like other followers of *Naturphilosophie*, he argued, that nature is animated and that there is an original unity or 'identity' of nature and mind which allows us to infer nature's laws from the laws of the mind and vice versa.

In 1839, Fechner had a nervous breakdown as the result of a depressive psychosis. He developed an aversion to food and also to light and experienced temporary blindness and complete prostration. Although he kept the title of a physics professor, he eventually lost his physics chair to Wilhelm Weber and was set on a pension by the university. After his recovery in 1846, he continued lecturing on diverse subjects, especially the mind-body problem, until 1875.

When his crisis was over, Fechner tried to come to terms with the two opposing tendencies of his thought, the strict mechanist mathematical physics on the one side and the romantic *Naturphilosophie* on the other. As a result of this, he developed a solution to the mind-body problem called 'psychophysical parallelism' or 'dual aspect theory' which became very popular among scientists in the 19th century. This solution is supposed to be compatible with science as well as with *Naturphilosophie* and it is central, both to Fechner's subsequent philosophical as well as scientific and mathematical work. According to this view, mind is not to be seen as a substance interacting with the body but as a special attribute of matter on which it is functionally dependent. In the same way as the appearance of an ordinary object, like the back and front of a coin, depends on the perspective of the viewer, so a person, as a body with a mental dimension, can be seen from the outside as well as from the inside. Mind and body are two different aspects of one and the same object. A person appears to outside viewers in another way than to herself or himself. As a consequence, it would not make any sense to say that the mind acts on the body or vice versa as it would be senseless to say that the back of the coin acts on its front when the coin is bent. Rather, if there is a change of a person it can be viewed in a mental as well as in a physical respect. Similarly, the bending of a coin results in a change of the coin's front and of its back at the same time. In thus rejecting causality as an appropriate category for the mind-body relation, Fechner thought he had shown psychophysical parallelism to be compatible with the principle of the conservation of energy. On the basis of his theory, Fechner founded the science of psychophysics, which became the starting point for experimental quantitative psychology.

Before it can be shown what all this has to do with statistics, we have to turn to another philosophical development. One of the most influential and powerful philosophical systems of the first half of the 19th century was that of G. W. F. Hegel. He claimed to have developed a logic which could explain history and nature as the necessary conceptual development of the idea on the way to self-knowledge. One of the most outspoken critics of this system was the Leipzig philosopher Christian Hermann Weisse, Fechner's closest friend. Weisse criticised Hegelian "panlogism" as not giving enough justice to the contingent and individual in nature and history. He argued that concrete reality is not the product of a logically necessary development of ideas as Hegel wanted it; there is something in it that transcends all necessity.

In two addresses of 1849, Fechner tried to show that taking Weisse's idea of indetermination seriously does not preclude the use of mathematics in science. He argued that mathematical descriptions only provide a general frame for natural phenomena not implying any necessity for the individual case, thus being compatible with an indeterminate behaviour on a finer level. He also claimed that in order to admit indeterminate events in nature one does not have to give up the causal law. This law only says that the same effect will recur if the same set of conditions obtains, but it does not preclude the emergence of new conditions in the course of time. This discussion might very well be the first expression of an indeterministic world-view.

The first major application of statistical methods by Fechner, and the first work, where his ideas on psychophysical parallelism and individual indeterminacy come together, are his *Elements of Psychophysics* of 1860. There, Fechner defined psychophysics as the "exact science of the functional or dependency relations between body and mind, or more general: between the bodily and the spiritual, physical and psychical, world." (Fechner 1860, I, 8) His goal was to measure sensations experimentally and thus to arrive at a quantitative science of psychophysics. The major result of his research was the so-called Weber-Fechner law which says that the intensity of sensation E increases in proportion to the logarithm of a stimulus R, or: $E = klogR$.

Fechner conceived of psychophysics as a fundimentally statistical enterprise. The rationale behind his reasoning seems to have been the following: If human beings are free in their actions and if mind and body are correlated in the way as conceived by psychophysical parallelism, then there will be an individual variation in the response of a subject to a physical stimulus. This response will be physical and manifest itself in a certain bodily reaction, but it will also be mental and express itself in a certain judgement. The fluctuations are not to be taken as erroneous deviations from the true value but as the free mode of reaction of the individual to a stimulus.

Among the three methods Fechner developed for measuring sensation is the "method of right and wrong cases". A subject had to lift a pair of weights, P and $P + D$, and to judge which seemed to be the heavier of the two. After n trials the ratio r/n of right answers to all trials was calculated. Fechner took the "measure of precision" h that appears in older formulations of the Gaussian law as an expression of the differential sensitivity of the subject, such that $2r/n - 1 = \theta(hD/2)$, where θ is the Gaussian law.

In 1878, Fechner published a paper where he developed the notion of the median. He later delved into experimental aesthetics and thought to determine the shapes and dimensions of aesthetically pleasing objects. He mainly used the sizes of paintings as his data base. In his *Vorschule der Aesthetik* of 1876 he used the method of extreme ranks for subjective judgements.

Fechner's most important contribution to statistics is his posthumously published book on the measurement of collectives, his *Kollektivmasslehre* of 1897. Fechner defined a 'collective' or 'collective object' as a collection of an indefinite number of individual objects, subject to random variation, and embraced under a single specific or generic concept. The main examples he treated are to be found in anthropology, zoology, botany, meteorology, aesthetics. The object of the enquiry is, as Fechner wrote,

> "the establishment, by mathematical proof and empirical verification, of a generalisation of Gauss's law of accidental variations, whereby the law is enabled to transcend the limits of symmetrical probability and comparative smallness of the positive and negative deviations from the arithmetical mean, and new relations of uniformity are brought to light." (Fechner 1897, vi)

He developed a set of constants which allowed to characterise different distributions and developed a two-sided asymmetric Gaussian law where the two branches are treated as if originating from two different distributions.

Fechner took the random variation of the collectives quite literally. He spoke of the "ideal laws of chance" which are realised in true collectives. Chance was for him an objective category and not just the expression of ignorance. He tried to design tests whereby variation due to factors other than chance could be detected by comparing the data under consideration to a random sequence.

Fechner's *Kollektivmasslehre* draws on several traditions. There is the moral statistics of the Belgian statistician and astronomer Adolphe Quetelet (q.v.) who was one of the first to investigate mass phenomena and to find numerical regularities in them. (Porter 1986) There is also the error theory of the mathematical astronomers Gauss (q.v.), Encke, Bessel and Hauber. And there is the tradition of the statistical bureaux of state administration. Much of Fechner's concept of a collective derives from the Württemberg state official Gustav Rümelin (1815-1889) who had distinguished between particulars that are typical of their genus and individuals that do not allow a straightforward inference as to the nature of the genus as a whole. The latter ones form a collective object and are the subject of statistics as the science of mass phenomena.

Fechner's *Kollektivmasslehre* was of immediate influence on many of his colleagues in Leipzig. The psychologists Gottlob Friedrich Lipps, Wilhelm Wirth and to some extent also Wilhelm Wundt used the new methods in psychophysics. Charles Edward Spearman who obtained his Ph.D. under Wundt extended Fechner's ideas and studied the correlation between magnitudes. The Leipzig astronomer and mathematician Heinrich Bruns (1848-

1919) soon gave a general solution to Fechner's problem of a mathematical representation of frequency distributions, the so-called Bruns-series (today called Gram-Charlier series) and tried to unify Fechner's theory of collectives with probability theory. His most illustrious student was Felix Hausdorff who carried this tradition further. (Girlich 1996) Bruns and Hausdorff, however, dropped Fechner's requirement of chance variation of the collective object, thus obscuring any trace of Fechner's indeterminism.

In Richard von Mises' (q.v.) theory of probability of 1919, however, this condition becomes one of two central requirements. As an axiomatic basis of his theory, von Mises postulated 1. the existence of the limiting values of the relative frequencies and 2. the randomness of the way how the attributes are mapped unto the elements of the collectives. Randomness is thereby defined as the invariance of the frequencies under any place selection - a criterion which clearly shows traces of Fechner's above mentioned test of homogeneity. Von Mises' theory gives a precise formulation of Fechner's basic intuitions. It marks the final defeat of the subjective Laplacean interpretation of probability, consolidates the frequency interpretation and conceives of probability as an empirical science of chance phenomena. One can only speculate what the course of statistics would have been if Fechner's work had been published before K. Pearson (q.v.) developed his biometrics in the early 1890s.

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